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METODI DI CALCOLO DEI FLUSSI DI POTENZA IN RETI RADIALI DISSIMMETRICHE

POWER FLOW CALCULATION METHODS FOR ASYMMETRICAL RADIAL NETWORKS

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Abstract

In some applications like power quality and safety analyses, it is of special interest to ascertain the power flow in the network. This is an issue that has become more important with the increasing of the distributed generation.

In this work a problem with a three-phase distribution network is analysed. Using the tool MATLAB, the radial network under analysis has been represented and studied. A system comprising 10 mini-pillars and 74 customers has been developed. Both the neutral wire and ground were included.

This work has the objective of recognising flaws and strengths of some approaches to the power flow calculation methods. Here reported are the results of what I could first learn, during a three months experience I made at the Dublin Institute of Technology in Ireland, and then more deeply analysed, once came back at the Department of Industrial Engineering of Padova in Italy.
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Introduction

The electrical system has been designed with the idea of centralised management and, in agreement with the type of plants so far more widespread (thermal and hydroelectric), unidirectional.

However, it is well known by now the importance of the possibilities renewable resources offer us and, thanks to the development of power electronics and the improvement of the transmission of energy, of the distributed generation.

The benefits obtainable through the use of energy from natural sources are known and belong mainly in the areas of renewability and low emissions. For these reasons, among the next targets of the European Union, measures regarding the use of these sources are included: increase to 20% by 2020 with a reduction of cost production and the gradual decrease in financial support.

On the one hand the integration of generated distribution in distribution networks leads clearly to significant benefits, on the other hand it forces a rethinking of the way of operation of the old networks. This is the reason why *smart grid* is now a common term.

For the regulation and the stability properties of the electric system the resolution of the problems of load flow is a topic of considerable importance. In case it was not possible to model the loads considering the absorption or injection of constant current, it would be necessary to face the problem by solving nonlinear equations. In order to avoid this type of calculations, preferring to deal with linear systems, iterative methods are called to intervene. Among them there are two that will be analysed and compared in this work.

The first one is the backward-forward method. This method presents very good features in terms of speed and solidity. The b-f sweep algorithm is very pioneer and most commonly used method for the power flow calculation of balanced and
unbalanced radial feeders. This is often used as a benchmark for comparison with other algorithms. However this algorithm was not designed to solve meshed networks[1].
Anyway it is thanks to its speed and strength that this method is the subject of considerable studies and improvements.

The second method involves the use of the complex admittance matrix - CAM. This approach enables each network component, such as lines, loads, generators and connections to be considered in a single complex matrix. As the one presented above, it is an iterative method. Here the power flow solution is reached with iterations in complex form without the need of real/imaginary decomposition. This procedure allows simple programming with any match package and has shown to have excellent convergence properties. The distinctive performance of this method is the high accuracy of the solutions also in ill-conditioned cases[3].

These procedures can be easily implemented in any commercially available math packages.
Chapter 1

Introduction to smart grids

Italian electric grid has historically been designed and built as an essentially one way passive network. The new power grid will make use of renewable energy resources and thus integrate into the electrical system differently sized plants, forming in this way the so-called distributed generation network - DG.

1.1 Distributed energy supply

At present time distributed generation is able to work with small quantities of energy, but a massive diffusion of this type of energy production would lead to a significant degradation of the efficiency and quality of distributed energy. Since the distributed generation systems will have many different characteristics and locations, a larger use of this type of generation represents a challenge in terms of control. It is also to consider that generation capacity from renewable energy floats widely being so dependent on local weather conditions, which are difficult to predict. Centralised control begins to appear more difficult when managed by the operator of the distribution network of energy.

A smart grid would re-design the network in order to manage micro-generation and the new bi-directional energy flow.

A smart grid is the set of an information network and of a power distribution network that allows to manage the electricity network in an intelligent way. This type of grid permits to combine information about the behaviours of suppliers and consumers. Its aim is to guarantee efficiency for the power distribution network and to lead to a more rational use of energy, minimising overloads and voltage variations around its nominal value.
So regarding control levels power smart grids must be very advanced, each device must be connected to the network to communicate and receive data in real time: a power grid littered with systems of monitoring and control. This is crucial in view of the advent of users or prosumers who buy but are also able to sell the electricity produced in-house, in an open market to large distributors as well as to small producers.
Chapter 2

Dublin, Ireland

The transmission system in Ireland is a meshed network of approximately 6500km of high voltage overhead lines, underground cables and over 100 transmission stations. The values of high voltage in Ireland are the same that can be found in Italy: 110kV, 220kV and 400kV.

Power is generated by power plants and wind farms throughout the country, utilising a variety of fuel or energy sources including gas, oil, coal, peat, hydro, wind and other sources such as biomass and landfill gas. All of the major generating plant feed into the national grid and power is transmitted nationwide. This design ensures that power can flow freely to where it is needed and that if one power station, power line or transmission station is non-operational, whether due to a fault, for maintenance or for any other reason, there are other options or routes available.

At the transmission stations power is transmitted from the grid, transformed into medium and low voltages, 38kV, 20kV and 10kV, and diverted into the lower voltage distribution system or directly to large industrial operations. The distribution system is separately managed by the Distribution System Operator (DSO), ESB Networks and brings power directly to Ireland’s domestic, commercial and industrial customers.
2.1 My work in Dublin

The main objective of my experience in Dublin was to recognise a way to develop an efficient and reliable simulation program to solve power-flow problems. It was also required that the program would work with large extended networks. Different methods were used to achieve the result, here they are presented in chronological order, according to my experience at the Institute.

Using MATLAB, the chosen radial network has been represented and studied.

2.1.1 About the considered network

The details of the section of the low voltage LV network, concerning a suburb of Dublin City, were acquired from the Irish Distribution Network Operator (DNO).

A system comprising 10 mini-pillars and 74 customers has been developed. Every consumer is supplied through a 10/0.4kV substation transformer. Each user has a single phase line. Both the neutral wire and ground were included.

Fig. 1 Radial part of Dublin’s network under consideration, with 9 mini-pillars and 74 consumers
The single-phase supplies at 230V and each consumer has a distinct earthing arrangement that complies to TNC-S.

Service cabling from mini-pillars to consumers is modelled as overhead (25/16 mm² concentric neutral), whereas, for the cabling from the substation transformer to the first mini-pillar and for each successive mini-pillar thereafter, 4-core, underground cabling is employed (either 185/70 mm² cross-linked polyethylene XLPE, or 70 mm² paper-insulated NAKBA)[8].

The earth electrode is connected to the installation’s main earth terminal MET and therefore to the DNO neutral. This arrangement provides the consumer with an earth terminal, which is connected to the neutral conductor of the system, thereby providing a low impedance path for the return of earth fault currents.
Chapter 3

The Backward-Forward method

When it is needed to find a solution for a non-linear problem, like in the case of the distribution network, iterative methods are the ones called to find an answer. At first, at the Department the solution was sought adopting a backward-forward method.

3.1 How it works

The Backward-Forward (B-F) is one of the techniques based on Ohm’s and Kirchhoff’s laws and refers to aforementioned methods. There is also another important family of what can be considered as classical processes that belongs to the iterative methods. The Newton-Raphson’s is indeed one of the most developed and usually chosen solution, since it requires a small number of iterations to reach the result almost independently from the size of the system to solve. Unlike it, the B-F needs many iterations to achieve the convergence. This might seem like a disadvantage, however the time required by the CPU (Central Processing Unit) for each iteration is shorter, so that the overall time spent to reach the final solution is much lower than the one needed using a different family of classical methods. The CPU time required at each iteration for the Newton-Raphson solution can be considerably high if it is necessary to proceed to the matrix’s inversion.

So this technique uses a sweep load flow algorithm that suits for radial distribution systems. The b-f method proceeds through four steps[2]:
1. the backward sweep uses Kirchhoff’s Voltage Law - KVL and the Kirchhoff’s Current Law - KCL, to obtain the voltage at each upstream bus, the calculation of the currents required by the loads and the lines shunt admittances, on the basis of the calculated or fixed values of nodal voltages;

2. the evaluation of the current (or power) flows in the branches composing the electrical system, starting from the terminal branches and going up to the source node;

3. the nodes voltages calculation, starting from the source node and proceeding to the terminal ones (forward sweep);

4. the verification of a convergence criterion; if it is satisfied the process stops, otherwise it restarts from the first step.

There are three main variations of the b-f method that differ depending on the type of electric quantities calculated at each iteration in the backward step[5]:

a) the current summation method, in which the branch currents are evaluated;

b) the power summation method, in which the power flows in the branches are evaluated;

c) the admittance summation method, in which, node by node, the driving point admittances are evaluated.
3.2 Representation of the line

Fig. 2 Model of the line

Designing the structure of a network for power-flow studies purposes, some particular attention must be given to the return current, that is the current that flows through the conductors and goes back to the grid through the ground, forming a closed circuit. This return current is due to unbalanced loads and nonlinear characteristics of the electrical equipment and may be larger than the phase currents if three-phase loads are unbalanced[6]. As the Carson’s paper suggests, the return current can be modelled by a conducting plane located at a certain depth below the earth surface.

A 5X5 matrix had been adopted for network representation, in order to consider three-phase wires, neutral wire and ground wire. It was assumed that in the considered section of the line the conductors are parallel to the ground.
\( Z_l \) is the line impedance of the \( l \) section of the line:

\[
\begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} & Z_{ag} \\
Z_{ab} & Z_{bb} & Z_{bc} & Z_{bn} & Z_{bg} \\
Z_{ac} & Z_{bc} & Z_{cc} & Z_{cn} & Z_{cg} \\
Z_{an} & Z_{bn} & Z_{cn} & Z_{nn} & Z_{ng} \\
Z_{ag} & Z_{bg} & Z_{cg} & Z_{ng} & Z_{gg}
\end{bmatrix}
\]

This matrix contains zeros at the corresponding row and column of any phase, neutral wire, or grounding, of the corresponding line section that does not exist.

The equation initially considered to create the elements of the \( Z_l \) matrix where the ones presented in Ciric’s paper\[2\]:

\[
Z_{ab} = j4\pi 10^{-4} f \ln \left( \frac{\sqrt{d_{ab}^2 + (h_a + h_b)^2}}{\sqrt{d_{ab}^2 + (h_a - h_b)^2}} \right) \left[ \frac{\Omega}{km} \right]
\]

\[
Z_{ag} = -\frac{1}{2} j4\pi 10^{-4} f \ln \left( \frac{\sqrt{\rho/f}}{h_a} \right) \left[ \frac{\Omega}{km} \right]
\]

\[
Z_{gg} = \pi^2 10^{-4} f - j 0.0386 \cdot 8 \cdot \pi 10^{-4} f + j4\pi 10^{-4} f \ln \left( \frac{2}{3.6195 \cdot 10^{-2}} \right) \left[ \frac{\Omega}{km} \right]
\]

\( r_a \) = resistance of phase \( a \) [\( \Omega \)]

\( h_a, h_b \) = heights of phase wires [m]

\( f \) = frequency [Hz]

GMR = Geometrical Mean Radius [m]

\( \rho \) = ground resistivity 100 [\( \Omega \)m]
Than the backward-forward technique works as it follows.

• nodal currents calculation:

\[
\begin{bmatrix}
I_{ja} \\
I_{jb} \\
I_{jc} \\
I_{jn} \\
I_{jg}
\end{bmatrix}
= \left( \begin{array}{c}
\frac{Z_{gi}}{Z_{nni} + Z_{gi}} \\
\frac{Z_{nni}}{Z_{nni} + Z_{gi}}
\end{array} \right)
\begin{bmatrix}
I_{ia} + I_{ib} + I_{ic} \\
I_{ia} + I_{ib} + I_{ic}
\end{bmatrix}
\]

where \(Z_{gi}\) and \(Z_{nni}\) are the ground and neutral mutual impedances respectively (\(Z_{gi}=Z_{gri}+Z_{ggi}\))

• backward sweep, section current calculation:

\[
\begin{bmatrix}
J_a \\
J_b \\
J_c \\
J_n
\end{bmatrix}
= - \begin{bmatrix}
I_a \\
I_b \\
I_c \\
I_n
\end{bmatrix}
+ \sum_{m \in M} \begin{bmatrix}
J_{ma} \\
J_{mb} \\
J_{mc} \\
J_{mn}
\end{bmatrix}
\]
• **forward sweep**, nodal voltage calculation and **correction**:

\[
\begin{bmatrix}
V_a^p \\
V_b^p \\
V_c^p \\
V_n^p
\end{bmatrix} = \begin{bmatrix}
V_a^p \\
V_b^p \\
V_c^p \\
V_n^p
\end{bmatrix} - \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\
Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\
Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\
Z_{na} & Z_{nb} & Z_{nc} & Z_{nn}
\end{bmatrix} \begin{bmatrix}
J_a \\
J_b \\
J_c \\
J_n
\end{bmatrix}
\]

Since the ground is considered as a conductor, the correction of the voltages of all the nodes with grounded neutral is carried out.

\[V_n = Z_{gr} \ast J_g\]

• **converge criteria**:

the difference between the solution now obtained and the one produced by the previous iteration is calculated. The process is interrupted when this gap results inferior to a set tolerance. If not, the structure will be repeated starting back from the node currents calculation, but this time using the voltage and currents flows obtained in the previous iteration.

**Flat start:** it is also imposed the initial voltage for the nodes to be equal to the root node voltage.

\[
\begin{bmatrix}
V_{ia} \\
V_{ib} \\
V_{ic} \\
V_{in} \\
V_{ig}
\end{bmatrix} = \begin{bmatrix}
V_{ref} \times e^{j120} \\
V_{ref} \times e^{-j120} \\
0 \\
0
\end{bmatrix}
\]

This type of structure permitted to obtain the required results and to provide a realistic analysis of the network.
It was observed anyway that to reach more accurate results the algorithm should be modified to include Carson-Clem’s equation.

3.3 Carson-Clem’s equations

Ciric’s methodology had the inconvenience of not considering the finite conductivity of the earth.

Carson’s equations permit to consider the influence of the earth resistance and of the currents that flow through it.

Since a distribution feeder is inherently unbalanced, every analysis that would show some precision should not make any assumptions regarding conductor sizes, the spacing between conductors, and transposition. In order to face this problem of accuracy, in his paper in 1926 Carson employed the image theory to develop equations that calculate the self-impedance with earth return and mutual impedances with common earth return for an arbitrary number of overhead conductors.

The image theory states that every conductor at a given distance above ground has an image conductor the same distance below ground.

Carson assumed the earth is an infinite, uniform solid with a flat uniform upper surface and a constant resistivity. Any end effects introduced at the neutral grounding points are not large at power frequencies, and were therefore neglected[9].

These equations can also be applied to underground cables.
The self impedance of a conductor $i$ would be:

$$z_{ii} = (r_i + 4 \cdot \omega \cdot P) + j2 \cdot \omega \cdot G \left( x_i + \ln \frac{S_{ii}}{\text{Radius}_i} + 2 \cdot Q \right)$$

While the mutual impedance between the conductor $i$ and a conductor $j$ is:

$$z_{ij} = 4 \cdot \omega \cdot P + j2 \cdot \omega \cdot G \left( \ln \frac{S_{ij}}{D_{ij}} + 2 \cdot \omega \cdot Q \right)$$

Expressed in ohm per miles [$\Omega$/mile] and where:

$$x_i = \ln \left[ \frac{\text{Radius}_i}{\text{GMR}_i} \right]$$
\[ P_{ij} = \frac{\pi}{8} - \frac{1}{3/2} k_{ij} \cos(\theta_{ij}) + \frac{k_{ij}^2}{16} \cos(2\theta_{ij}) \cdot \left( 0.6728 + \ln \frac{2}{k_{ij}} \right) \]

\[ Q_{ij} = -0.0386 + \frac{1}{2} \cdot \ln \frac{2}{k_{ij}} + \frac{1}{3/2} k_{ij} \cos(\theta_{ij}) \]

\[ k_{ij} = 8.565 \times 10^{-4} \cdot S_{ij} \cdot \sqrt[3]{\frac{f}{\rho}} \]

**Radius;** \(i\) = conductor radius in feet  
**GMR;** \(i\) = Geometric Mean Radius of conductor \(i\) in feet  
**zi;** = self impedance of conductor \(i\) in \(\Omega/mile\)  
**zij;** = mutual impedance between conductors \(i\) and \(j\) in \(\Omega/mile\)  
**ri;** = resistance of conductor \(i\) in \(\Omega/mile\)  
**\(\omega = 2\pi f\);** = system angular frequency in radians per second  
**\(G = 0.1609344 \times 10^{-3}\);** \(\Omega/mile\)  
**RD;** \(i\) = radius of conductor \(i\) in feet  
**\(f\);** = system frequency in Hertz  
**\(\rho\);** = resistivity of earth in \(\Omega\)-meters  
**Dij;** = is the distance between the axis of the conductor with respect to the \(i\)-th and \(j\)-th conductor;

Carson’s original equations were described in Ohms/mile, but the subsequent cabling considerations that will be discussed here are described as Ohms/kilometre.
Anyway since the use of there equations was troublesome, some approximations were made in deriving the modified Carson’s equations. These approximations were firstly developed by Carson himself, involving the terms associated with $P_{ij}$ and $Q_{ij}$ by using only the first term of the variable $P_{ij}$ and the first two terms of $Q_{ij}$. So $P_{ij}$ and $Q_{ij}$ were defined as constants correction terms.

The technique was not met with a lot of enthusiasm because of the tedious calculations that would have to be done on the slide rule and by hand. With the advent of the digital computer, Carson’s equations have become widely used.

Anderson also derived equations to describe the self and mutual impedances of the lines. He considers the Carson's line as a single return conductor with a self GMD of 1 foot (or 1 meter), located at a distance (unit length) above/below the over head or under ground line, depending on the situation. This said distance is a function of the earth resistivity. In his description of the cable impedances, Anderson employs the concept of hypothetical return path of the earth current and is a function of both earth resistivity and frequency.

Through these approximations new Carson’s equations where derived, here are presented the Carson-Clem’s equations, which are the ones that were finally used in the program.

Each conductor’s impedances are obtained according to Carson's formulas with the following assumptions: the conductors are parallel to each other and the earth is homogenous within a span.

The distance of the conductors from the centroid of the return currents in the ground is defined as:
The equivalent return conductor resistance into the soil depends on the soil resistivity:

\[ D_e = \frac{1.852}{\alpha} = 660, \sqrt{\frac{\rho}{f}} \, [m] \]

\[ \alpha = \sqrt{\frac{\mu_0 \rho}{f}} \, \left[ \frac{1}{m} \right] \]

Since the earth resistance is here to be considered, it would be:

\[ R_{\text{earth}} = \pi^2 f + 10^{-4} \left[ \frac{\Omega}{km} \right] \]

So that the equations describing the self and mutual impedances now are:

\[ Z_{ii} = R_{ii} + R_{\text{earth}} + j4\pi f \times 10^{-4} \times \ln \left( \frac{2D_e}{d} \right) \left[ \Omega/km \right] \]

\[ Z_{ij} = Z_{ji} = R_{\text{earth}} + j4\pi f \times 10^{-4} \times \ln \left( \frac{D_e}{d_{ij}} \right) \left[ \Omega/km \right] \]

R_{ii}: is the kilometric resistance of the conductor, expressed in [\Omega / km]

These expressions can be applied when the distance between the conductors results less than 15% of the D_e equivalent distance back into the soil. It is considered that if this condition is met, the error in determining the impedance is less than 2.5%. Thanks to these equations it is now possible to derive the matrix of the longitudinal impedances for both airlines for cable lines with a number n of conductors.
Considering a line scheme like this:

The correspondent impedance matrix would be:

\[
[Z_{\text{primitive}}] = \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} & Z_{an1} & Z_{an2} & Z_{an3} \\
Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn1} & Z_{bn2} & Z_{bn3} \\
Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn1} & Z_{cn2} & Z_{cn3} \\
Z_{n1a} & Z_{n1b} & Z_{n1c} & Z_{n1n1} & Z_{n1n2} & Z_{n1n3} \\
Z_{n2a} & Z_{n2b} & Z_{n2c} & Z_{n2n1} & Z_{n2n2} & Z_{n2n3} \\
Z_{nma} & Z_{nmb} & Z_{nmc} & Z_{nmn1} & Z_{nmn2} & Z_{nmn3}
\end{bmatrix}
\]

For most applications this primitive impedance matrix needs to be reduced. One standard method of reduction is the Kron reduction. The assumption is made that the
The Kron reduction method applies Kirchhoff’s voltage law to the circuit. By this simplification the matrix becomes:

\[
\begin{bmatrix}
V_{abc} \\
V'_{abc} \\
V_{ng} \\
V'_{ng}
\end{bmatrix} = \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\
Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\
Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\
Z_{na} & Z_{nb} & Z_{nc} & Z_{nn}
\end{bmatrix}
\begin{bmatrix}
Z_{ij} & Z_{in} & I_{abc} \\
Z_{nj} & Z_{nn} & I_{n}
\end{bmatrix}
\]

Because the neutral is grounded, the voltages \(V_{ng}\) and \(V'_{ng}\) are equal to zero. It means that the equation written above can now be re-written as:

\[
[V_{abc}] = [V'_{abc}] + [Z_{ij}] [I_{abc}] + [Z_{in}] [I_{n}]
\]

Which allows to solve the equation for:
\[ I_n = - Z_{nn}^{-1} * Z_{nj} * I_{abc} \]

That substituted in the previous equation, gives:

\[ [Z_{abc}] = [Z_{ij}] + [Z_{in}] * [Z_{nn}]^{-1} * [Z_{nj}] \]

that is the final form of the Kron reduction technique. The final phase impedance matrix becomes:

\[
\begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix}
\]

The network cabling was then modelled by considering the modified Carson equations also in terms of the electromagnetic coupling that is developed between parallel lines.

### 3.4 Representation of the generators

The use of the wind turbine generators commonly involves the use of induction motors, which differs completely from the other forms of conversion of electrical energy in the distributed generation, as induction generators require reactive power for their operation. In this case, generation is modelled as constant power source with a power factor of 0.95.
3.5 Representation of the loads

Depending on their type of the load its modelling changes. For the second step of the backward-forward method this is a crucial point because it influences the nodal currents calculation. In this work it has been considered to have a three-phase star or phase to ground connection.

The diagram to refer to is this one:

![Diagram](image)

Fig. 4 Load modelling scheme for three-phase star or phase to ground connection

3.6 Different implementations

Here are reported the scripts of the two different approaches to the construction of the line impedances.

3.6.1 Ciric’s scheme

As it can be seen, in the first script proposed the earth resistance $R_e$, and so the distance of the conductors $D_e$, are neglected.
%% DEFINING THE PRIMITIVE (partitioned) Z-MATRIX

\[ z_{aa} = ri_\text{line} + i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{2 \cdot h_a}{GMR_\text{line}}) \]
\[ z_{bb} = ri_\text{line} + i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{2 \cdot h_b}{GMR_\text{line}}) \]
\[ z_{cc} = ri_\text{line} + i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{2 \cdot h_c}{GMR_\text{line}}) \]
\[ z_{nn} = ri_n + i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{2 \cdot h_n}{GMR_n}) \]
\[ z_{ab} = i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{\sqrt{d_{ab}^2 + (h_a + h_b)^2}}{\sqrt{d_{ab}^2 + (h_a - h_b)^2}}) \]
\[ z_{ac} = i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{\sqrt{d_{ac}^2 + (h_a + h_c)^2}}{\sqrt{d_{ac}^2 + (h_a - h_c)^2}}) \]
\[ z_{an} = i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{\sqrt{d_{an}^2 + (h_a + h_n)^2}}{\sqrt{d_{an}^2 + (h_a - h_n)^2}}) \]
\[ z_{na} = z_{an} \]
\[ z_{ba} = z_{ab} \]
\[ z_{bc} = i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{\sqrt{d_{bc}^2 + (h_b + h_c)^2}}{\sqrt{d_{bc}^2 + (h_b - h_c)^2}}) \]
\[ z_{bn} = i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{\sqrt{d_{bn}^2 + (h_b + h_n)^2}}{\sqrt{d_{bn}^2 + (h_b - h_n)^2}}) \]
\[ z_{nb} = z_{bn} \]
\[ z_{ca} = z_{ac} \]
\[ z_{cb} = z_{bc} \]
\[ z_{cn} = i \times 4 \pi \times 10^{-4} \times f \times \log(\frac{\sqrt{d_{cn}^2 + (h_c + h_n)^2}}{\sqrt{d_{cn}^2 + (h_c - h_n)^2}}) \]
\[ z_{nc} = z_{cn} \]

%% Primitive Matrix

\[ \text{Zabcng} = [z_{aa} \ z_{ab} \ z_{ac} \ z_{an}; \ z_{ba} \ z_{bb} \ z_{bc} \ z_{bn}; \ z_{ca} \ z_{cb} \ z_{cc} \ z_{cn}; \ z_{na} \ z_{nb} \ z_{nc} \ z_{nn}] \times (c_{\text{length}}/1000); \]

end

3.6.2 With Carson-Clem’s equations

This different way of representing the line impedances influences the values that will be considered for the voltages.
\[
\begin{align*}
zaa &= (ri\_line + RE + i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log(De/GMR\_line)); \\
zbb &= (ri\_line + RE + i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log(De/GMR\_line)); \\
zcc &= (ri\_line + RE + i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log(De/GMR\_line)); \\
znn &= (ri\_n + RE + i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log(De/GMR\_n)); \\
\end{align*}
\]

\[
\begin{align*}
zab &= i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log((\sqrt{dab^2 + (ha + hb)^2}) / (\sqrt{dab^2 + (ha - hb)^2})); \\
zac &= i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log((\sqrt{dac^2 + (ha + hc)^2}) / (\sqrt{dac^2 + (ha - hc)^2})); \\
zan &= i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log((\sqrt{dan^2 + (ha + hn)^2}) / (\sqrt{dan^2 + (ha - hn)^2})); \\
zna &= zan; \\
zba &= zab; \\
zbc &= RE + i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log(De / (\sqrt{dbc^2 + (hb - hc)^2})); \\
zbn &= RE + i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log(De / (\sqrt{dbn^2 + (hb - hn)^2})); \\
znb &= zbn; \\
zca &= zac; \\
zcb &= zbc; \\
zcn &= RE + i \cdot 4\pi \cdot (10^{-4}) \cdot f \cdot \log(De / (\sqrt{dcn^2 + (hc - hn)^2})); \\
znc &= zcn;
\end{align*}
\]

%%                          Primitive Matrix

\[
\begin{align*}
Zabcng &= [zaa \ zab \ zac \ zan; \\
\ & \ \ \ \ \ \ zba \ zbb \ zbc \ zbn; \\
\ & \ \ \ \ \ \ zca \ zcb \ zcc \ zcn; \\
\ & \ \ \ \ \ \ zna \ znb \ znc \ znn] \cdot (c\_length/1000); \\
\end{align*}
\]

end

Since the criteria at the basis of the convergence is the difference between two voltages, when these values changes, the range for the reaching of the solution becomes too big to permit the program to stop after a reasonable number of iterations.

It does not mean that the modified program is wrong. On the contrary, this attempt of changing the characteristic of the line has highlighted the limits of the b-f technique, which where there from the beginning.
3.7 Which is the actual issue

Cable models are very influential in deriving any power flow solution aiming to derive accurate voltage profile results.

Using Kron’s reduction in fact, the information regarding the neutral voltages gets lost, since this value is forced to zero along the line. This type of imposition results much stronger as the network starts to present increasing levels of unbalancing.

It is thanks to that that the iteration could reach a solution, while it would be impossible when including the earth connection. By this approximation the network is changed in structure, considering the earth connection only at the substation (like in Italy), but not at the bus nodes (like in Ireland).

Including the Carson-Clem’s equations the converging criteria of the developed network does not permit to reach a solution.

It was initially supposed it was because of the criteria for applying the formulas had not been respected. So it was verified that also in this case the distance between the conductors resulting less than 15% of the equivalent distance of the return conductor into the soil $D_e$. Anyway the distance was proved appropriate.

The correction that Carson introduced to taken into account the finite value of the resistivity of the soil, for the calculation of the line impedances, can indeed result as the predominant term of the equation. In facts, referring to the scripts here reported, it results that $D_e$ can be much bigger that $h_a$, with predictable results changes.

When Ciric’s equation are considered, this fact is neglected as for the $R_e$ and $D_e$.

The impossibility for this algorithm to converge is due to the fact that, since the convergence depends on a range of voltages, using these new formulas a greater voltage variation is now introduced. The backward-forward method no longer suits.

Anyway the most important aspect to be considered is the chosen configuration for the line, which includes multi-grounded neutral. This is a key point in the reach of
the solution. What indeed does not permit to prefer the backward-forward model in this case is that with Ciric’s approach the voltage in the neutral tends to show very little variations, remaining almost constant. This type of condition is in this program forced, ignoring the earth loads and pillars connections that are characteristic of the Irish network that present the multi-grounded neutral configuration.

It is also the reason why, when the same script was applied to the Italian network, where the neutral is grounded only in the substations, the program including the Carson-Clem variation worked.
Chapter 4

Analysis of the Network Structure

The two systems presented above have been manipulated to obtain smaller and easier to manage networks, reducing the pillars from 9 to 2, changing the numbers of loads as well. This operation was made in order to permit a deeper investigation of the backward-forward method and so to achieve to find, if not a solution, a reason for the not proper working of Ciric’s approach when including the Carson-Clem’s equations. Here in this section are reposted the result obtained ignoring the earth connection.

4.1 B-F algorithm structure

Here are reported the most important functions and variables in the program, for a better comprehension of its development. As it can be seen in Appendix A, the program proposes two different ways to consider the lines, including or excluding the earth connection. It is in fact this type of connection that leads to more uncertainties and has in this way been highlighted.

In this analysis the parameters regarding the earth will be included.

Starting from the main program, here are presented the scripts, trying to give them an order meant to facilitate an immediate understanding.
4.1.1 Model_Entire_Network

- the system parameters are defined, such as power and voltage

\[
\begin{align*}
V_{\text{line}} &= 415; \\
\text{Sys}_\text{MVA}_\text{base} &= 1e+6; \\
z_{\text{base}} &= \frac{V_{\text{line}}^2}{\text{Sys}_\text{MVA}_\text{base}};
\end{align*}
\]

- fundamental sub-programs are called. In here there are the:

  - System_Z, to define the network impedances
  - Inital_Voltage, to assign the flat start to all system’s buses
  - Customer_Load_With_EarthElectrode, deriving the consumers loads from the .txt file containing the active and reactive parameters for each bus
  - Branch_Current, evaluating branch and pillars’ currents

- the loop flow is initialised for the calculation of voltages and currents

```matlab
for ii=1:369
    V_Diff=V_3;  ii
    Voltage_Update
    if max(abs(V_Diff-V_3))<0.00001
        break
    end
    Customer_Load_With_EarthElectrode
    Branch_Current
    V_3
end
```

- the RESULTS are called out

4.1.2 Sys_Load

Active and reactive powers P and Q are uploaded for each bus from a Load.txt file, together with the related voltages from the Customer_Load_With_EarthElectrode file.
This program calculates loads S power and provides loads current for each phase at each bus.

```matlab
function [Sabc, Iabcng] = Sys_Load(bus, V)

Sabc=[Load(row_ID,2)+j*Load(row_ID,3) Load(row_ID,4)+j*Load(row_ID, 5) Load(row_ID,6)+j*Load(row_ID,7)];

Ia=conj(Sabc(1))./(conj(V(1)-V(4)));
Ib=conj(Sabc(2))./(conj(V(2)-V(4)));
Ic=conj(Sabc(3))./(conj(V(3)-V(4)));

In=-(Ia+Ib+Ic)+(V(4))/Rcons;
```

### 4.1.3 Branch_Current

Gives the sum of loads currents, neutral currents included, to obtain pillar currents, updating them at each node.

```matlab
%Pillar B
Pillar=1;
I_46=IL_46;I_63=IL_63;I_75=IL_75;I_59=IL_59;I_26=IL_26;I_120=IL_120;
I_2=I_46+I_63+I_75+I_59+I_26+I_120+I_3;
I=I_2; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar,I,V_2,Rg_pillar);
I_2=I_Pillar;
% SUBSTATION A
I_1=I_2;
```

### 4.1.4 Pillar_Current

The sum of loads currents calculated in the Branch_Current does not gives the pillar current. It is in fact necessary to consider the pillar earth connection through the inclusion of the current due to the voltage trop in Rg_Pillar.

```matlab
function [I_Pillar]= Pillar_Current(Pillar, I,V,Rg_pillar)
    Pillar;
```
\[ \text{In} = I(4) + \frac{(V(4))}{R_{g\_pillar}}; \]

\[ I\_\text{Pillar} = [I(1); I(2); I(3); \text{In}]; \]

### 4.1.5 Voltage Update

From the Initial Voltages assigned for the flat start, voltages at each bus and pillar are updated considering the line impedances and the calculated currents.

```matlab
%-------- SOURCE ---------%
ZE = 0.00001;
IE = -(I_1(1) + I_1(2) + I_1(3) + I_1(4));
V_1(4) = IE * ZE;
V_1 = [ (1 + 0i) + V_1(4); (1*exp(-j*120*pi/180)) + V_1(4); (1*exp(-j*240*pi/180)) + V_1(4); V_1(4) ]; % in pu

%-------- Mini-pillar A1 ---------%
V_2 = (V_1 - (z_{A1}I_{2}));
[V2\_unbalance] = Voltage\_Unbalance(V_2);
V_{120} = V_2 - (z_{A1}C120*1_{120});
V_{26} = V_2 - (z_{A1}C26*1_{26});
V_{59} = V_2 - (z_{A1}C59*1_{59});
V_{75} = V_2 - (z_{A1}C75*1_{75});
V_{63} = V_2 - (z_{A1}C63*1_{63});
V_{46} = V_2 - (z_{A1}C46*1_{46});
```

The iterative method goes up and down the network to update the V and I parameters.

### 4.2 Comparison with Ciric’s initial formulations

In his article “Power flow in distribution networks with earth return, Electrical Power and Energy Systems 26 (2004)”, Ciric provided a solution for the power flow problem using the backward-forward technique. From these studies the researches for an adapted configuration to Dublin’s network started.
To evaluate the DIT proposed program, it is then important to compare it from where it was firstly created.

The crucial steps for the script development are here reported in association with Ciric’s ideas.

### 4.2.1 Nodal current calculation

The calculation of the currents is represented in the script by the formulas:

\[
I_a = \frac{\text{conj}(S_{abc}(1))}{\text{conj}(V(1)-V(4))};
I_b = \frac{\text{conj}(S_{abc}(2))}{\text{conj}(V(2)-V(4))};
I_c = \frac{\text{conj}(S_{abc}(3))}{\text{conj}(V(3)-V(4))};
\]

\[R_{\text{cons}} = 0.15;\]

\[I_n = -(I_a + I_b + I_c) + (V(4))/R_{\text{cons}};\]

Which differ from the equation proposed by Ciric by the exclusion of the correction admittance of all shunt elements at node \( Y_{ia} \ Y_{ib} \ Y_{ic} \ Y_{in} \), anyway the result will not be effected from this choice.

\[
\begin{bmatrix}
I_a^{(k)} \\
I_b^{(k)} \\
I_c^{(k)} \\
I_g^{(k)}
\end{bmatrix}
= \begin{bmatrix}
(S_{ia}/Y_{ia})^{(k-1)} & 0 & 0 & 0 \\
0 & (S_{ib}/Y_{ib})^{(k-1)} & 0 & 0 \\
0 & 0 & (S_{ic}/Y_{ic})^{(k-1)} & 0 \\
-(I_a^{(k)} + I_b^{(k)} + I_c^{(k)}) & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
Y_{ia} \\
Y_{ib} \\
Y_{ic} \\
0
\end{bmatrix}
\begin{bmatrix}
V_i^{(k-1)} \\
V_i^{(k-1)} \\
V_i^{(k-1)}
\end{bmatrix}
\]

### 4.2.2 Backward sweep

The script represents here Ciric’s structure, starting from the last line section and moving towards the root node.
4.2.3 Forward sweep

Starting from the first layer and moving towards the last layer, the voltage at node \( i \) is:

\[
\begin{bmatrix}
J_{la}^{(k)} \\
J_{lb} \\
J_{lc} \\
J_{lg}
\end{bmatrix}^{(k)} = -\begin{bmatrix}
J_{ja}^{(k)} \\
J_{jb} \\
J_{jc} \\
J_{jg}
\end{bmatrix}^{(k)} + \sum_{m \in M} \begin{bmatrix}
J_{ma}^{(k)} \\
J_{mb} \\
J_{mc} \\
J_{mg}
\end{bmatrix}^{(k)}
\]

4.2.4 Converge Criteria

The converge criteria for the program developed at the DIT is based on a voltage range tolerance:

\[
\begin{align*}
&\text{for } i=1:4000 \\
&\quad V_{\text{Diff}}=V_5; \quad i \quad \% \text{ Convergence parameter} \\
&\quad \text{Voltage Update} \\
&\quad \quad \text{if } \max(\text{abs}(V_{\text{Diff}}-V_5))<0.00001 \quad \% \text{ Convergence criteria} \\
&\quad \quad \quad \text{break}
\end{align*}
\]
while in Ciric’s paper the convergence is defined depending on power S mismatches:

$$\Delta S_{ia}^{(k)} = V_{ia}^{(k)} (I_{ia}^{(k)})^* - Y_{ia}^* |V_{ia}^{(k)}|^2 - S_{ia}$$

$$\Delta S_{ib}^{(k)} = V_{ib}^{(k)} (I_{ib}^{(k)})^* - Y_{ib}^* |V_{ib}^{(k)}|^2 - S_{ib}$$

$$\Delta S_{ic}^{(k)} = V_{ic}^{(k)} (I_{ic}^{(k)})^* - Y_{ic}^* |V_{ic}^{(k)}|^2 - S_{ic}$$

$$\Delta S_{ig}^{(k)} = V_{ig}^{(k)} (I_{ig}^{(k)})^*$$

If the real or imaginary part of any of the power mismatches is greater than a convergence criterion, steps 1, 2 and 3 are repeated until convergence is achieved.

4.3 Modified network - Obtained results

The algorithm was modified, reducing the number of the pillars under consideration, obtaining a simpler system to evaluate with less variable to compare, in order to facilitate the analysis to make. The objective of this modification was in fact to find and compare voltages values at different points of the network, to evaluate the variables’ evolutions in different contexts, sometimes excluding some loads.

4.3.1 Source -> Pillar B -> Pillar C

The first step was to modify the structure of the program to consider, together with the source, only two other pillars and all the connected loads. So three buses were analysed:

1) SUBSTATION - SOURRE - A
2) PILLAR B - that feeds loads - 1, 2, 3, 4, 5, 6

3) PILLAR C - that feeds loads - 7, 8, 9, 10, 11, 12, 13

Here in the Tables Voltages and Currents are reported.

**REDUCTION TO SOURCE + 2 MINI-PILLARS**

**RESULTS**

**Neutral Voltage**

<table>
<thead>
<tr>
<th>Bus#</th>
<th>Neutral</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V- Neutral</td>
</tr>
<tr>
<td>Source</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>0.2045</td>
</tr>
<tr>
<td>B</td>
<td>0.4015</td>
</tr>
</tbody>
</table>

Tab. 1

**Neutral Current**

<table>
<thead>
<tr>
<th>Bus#</th>
<th>Neutral</th>
<th>Neutral</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I-Magntude</td>
<td>I-Phase</td>
</tr>
<tr>
<td>Source</td>
<td>-0,0216</td>
<td>0,0058</td>
</tr>
<tr>
<td>A</td>
<td>-0,0216</td>
<td>0,0058</td>
</tr>
<tr>
<td>B</td>
<td>0,0165</td>
<td>0,0076</td>
</tr>
</tbody>
</table>

Tab. 2
<table>
<thead>
<tr>
<th>Line</th>
<th>Bus#</th>
<th>Pillar Currents</th>
<th>Pillar Voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1</td>
<td>0.0002</td>
<td>0.0117</td>
<td>0.0374</td>
</tr>
<tr>
<td>Line 2</td>
<td>0.0216</td>
<td>0.0374</td>
<td>0.0374</td>
</tr>
<tr>
<td>Line 3</td>
<td>0.0374</td>
<td>0.0374</td>
<td>0.0374</td>
</tr>
</tbody>
</table>

Reduction to Source + 2 Mini-Pillars
REDUCTION TO SOURCE + 2 MINI-PILLARS

Loads 2 and 5 of Line 2 and 9 and 10 of Line 3 Excluded

RESULTS

Neutral Voltage

<table>
<thead>
<tr>
<th>Bus#</th>
<th>Neutral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0,0787</td>
</tr>
<tr>
<td>C</td>
<td>0,2840</td>
</tr>
</tbody>
</table>

Tab. 5

Neutral Current

<table>
<thead>
<tr>
<th>Bus#</th>
<th>Neutral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>0,0108</td>
</tr>
<tr>
<td>B</td>
<td>0,0108</td>
</tr>
<tr>
<td>C</td>
<td>0,0096</td>
</tr>
</tbody>
</table>

Tab. 6
### Table 8

<table>
<thead>
<tr>
<th>Bus</th>
<th>Line 1</th>
<th>Line 2</th>
<th>Line 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.00116</td>
<td>0.00201</td>
<td>0</td>
</tr>
<tr>
<td>0.0199</td>
<td>0.0111</td>
<td>0.02</td>
<td>0.00117</td>
</tr>
<tr>
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<td>0.02</td>
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</tr>
<tr>
<td>Source</td>
<td>0.0117</td>
<td>0.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Table 7

<table>
<thead>
<tr>
<th>Bus</th>
<th>Line 1</th>
<th>Line 2</th>
<th>Line 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>119.9847</td>
<td>239.4131</td>
<td>239.0324</td>
<td>239.2015</td>
</tr>
<tr>
<td>119.9873</td>
<td>239.4325</td>
<td>239.0141</td>
<td>239.4739</td>
</tr>
<tr>
<td>0.866</td>
<td>0.5</td>
<td>0.866</td>
<td>0.5</td>
</tr>
<tr>
<td>Source</td>
<td>i</td>
<td>i</td>
<td>i</td>
</tr>
</tbody>
</table>

### Pillar Voltages

Loads 2 and 5 of line 2 and 9 and 10 of line 3 excluded.

Reduction to source + 2 mini-pillars.
Chapter 5

The Complex Admittance Method

5.1 Why another method investigation

The backward-forward method has some positive peculiarities and thanks to its speed and solidity this method is the subject of considerable study and improvements. It has been demonstrated anyway that it is not always appropriate to any type of network, especially when applying Carson - Clem’s formulas to a network that is structured so that each consumer has an earth electrode connection to earth.

On the other hand there were still some researches I had to do on regards of the CAM method, to clarify it’s proprieties and strengths.

Once the experience at the DIT was over, my researches regarding the CAM method needed to be deepened in order to provide an adequate comparison between the two models. In this case in particular I had to look for other theoretical references, as my work was initially mainly based on the article [10].

5.2 New network representation

Most of the methods have been developed to study transmission systems. Over the years variations of the Newton method (such as the fast decoupled method) have become the most widely used. However, even though the Newton-Raphson
technique is still a valid choice in the resolutions of the transmission networks, it can not be directly applied to the distribution networks because of some features of the latter distribution networks that need a different approach for its load flow analysis. These characteristics are[4]:

• high ratio $R/X$ of the feeder;
• asymmetry of the network;
• imbalance of loads;
• high number of nodes.

Thanks to the approach here presented, it is anyway possible to achieve a power flow solution also in those systems with high $R/X$ ratio in some lines and in situations close to voltage collapse.

The procedure is sufficiently robust to facilitate a multi-conductor asymmetrical network analysis. The methodology was originally developed for computing electromagnetic coupling of complex conductor geometries, incorporating network structure, load, generation and earthing elements.

It is thus based on the formal possibility to represent both loads and generators, except for the slack-bus, by shunt elements, which will be included in a nodal admittance matrix.
So the network can be considered as divided into two parts:

- a passive network, incorporating branch admittances;
- an active network, in which the shunt elements, load and generators, are represented as admittances.

The system may be thus thought as passive and can then be activated by the voltage phasor applied to the slack bus. The adjustment is achieved by injecting in parallel with the shunt admittances suitable correction currents with no modification of the initial admittance values[3].

If no generation is present, loads and generators can be represented as constant admittances. In this way, shunt and network branch admittances can be combined to form a complete network representation. In the same way the earthing network, as it is facilitated at network nodes, can also be included.

The first step in developing the mathematical model describing the power flow in the network is the formulation of the bus admittance matrix.
5.3 The admittance matrix

Includes branch and shunt elements and expresses the relationship between the currents and the voltages.
It is a symmetric matrix along the leading diagonal, which result in an upper diagonal nodal admittance matrix. The diagonal elements are the self-admittances at the nodes and the other components are the mutual admittances.

Now as an example are here considered three three-phase balanced loads fed by a three-phase line. The nominal loads’ powers are respectively $N_{N1}$, $N_{N2}$, $N_{N3}$.

![Fig. 6 Three-phase line with three loads connected](image)

Being $V_N$ the phase voltage of the system, each user can be represented with an admittance $Y_{us}$, so that:

$$
Y_{us} = \frac{N_{Ns}}{V_N^2} \quad (s = 1, 2, 3)
$$

Assuming now the symmetry of the network and of the phase voltages, the circuit can be studied through its single phase direct sequence representation.
Fig. 6 Equivalent circuit of a three-phase line with three connected loads considering the loads as admittances.

Considering now to know the star voltages $E_A$ and $E_B$ imposed at the ends of the circuits and applying the principle of superposition, the currents can be expressed as a function of the voltages and the admittances. A matrix can be constructed, which is the actually $Y$ admittance matrix:

$$
\begin{align*}
\dot{I}_A &= \begin{bmatrix} Y_{AA} & Y_{AB} & Y_{A1} \\ Y_{BA} & Y_{BB} & Y_{B1} \\ Y_{1A} & Y_{1B} & Y_{11} \end{bmatrix} \begin{bmatrix} \dot{E}_A \\ \dot{E}_B \\ \dot{E}_1 \end{bmatrix} \\
\dot{I}_B &= \begin{bmatrix} Y_{A2} & Y_{A3} \\ Y_{B2} & Y_{B3} \\ Y_{2A} & Y_{2B} \end{bmatrix} \begin{bmatrix} \dot{E}_A \\ \dot{E}_B \\ \dot{E}_1 \end{bmatrix} \\
\dot{I}_1 &= \begin{bmatrix} Y_{A1} \\ Y_{B1} \\ Y_{11} \end{bmatrix} \begin{bmatrix} \dot{E}_A \\ \dot{E}_B \\ \dot{E}_1 \end{bmatrix} \\
\dot{I}_2 &= \begin{bmatrix} Y_{A2} \\ Y_{B2} \\ Y_{21} \end{bmatrix} \begin{bmatrix} \dot{E}_A \\ \dot{E}_B \\ \dot{E}_1 \end{bmatrix} \\
\dot{I}_3 &= \begin{bmatrix} Y_{A3} \\ Y_{B3} \\ Y_{31} \end{bmatrix} \begin{bmatrix} \dot{E}_A \\ \dot{E}_B \\ \dot{E}_1 \end{bmatrix}
\end{align*}
$$

In a real line, this $n \times n$ matrix, where $n$ is the number of buses in the system, is constructed by the admittances of the equivalent circuit elements of the segments making up the power system.

Referring closely to the components that build the matrix and its meanings, the system segments in which the matrix can be divided, are represented by a
combination of shunt elements, connected between a bus and the reference node, and series elements, connected between two system buses. Each of these segments can be represented by a π-circuit.

Cascade connections of multiphase π-circuits can be easily used to model each span and, consequently, the full length of the line without any limits in the number of phase conductors.

5.3.1 Shunt elements

![Diagram](image)

Fig.7 Representation of the network as splitted into Passive and Shunt parameters

Line and shunt admittances can be combined to represent the entire network. Shunt components’ equations are here reported.

- For the loads’ admittances the calculation is obtained by an equivalent shunt impedance in cascade with the last cell of the system obtained:

\[ Y_{Load} = \frac{S_{Load(i)}}{\left| U_{Load(i)} \right|^2} \]
while the generators admittances are:

\[ Y_{\text{Load}} = S_{\text{Gen}(i)} / |U_{\text{Gen}(i)}|^2 \]

### 5.3.2 Branch elements

The branch elements matrix is constructed on the basis of bus-to-bus connection. An overhead four-wire grounded neutral distribution line segment will result in a 4 \( \times \) 4 matrix. If it is considered an underground grounded neutral line segment consisting of three concentric neutral cables, the resulting matrix will be 6 \( \times \) 6.

Since the admittance is the inverse of the impedance, \( Y = Z^{-1} \), the primitive impedance matrix for a three-phase line with \( m \) neutrals will be of the form[9]:

\[
\begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} & Z_{an1} & Z_{an2} & Z_{an3} \\
Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn1} & Z_{bn2} & Z_{bn3} \\
Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn1} & Z_{cn2} & Z_{cn3} \\
Z_{n1a} & Z_{n1b} & Z_{n1c} & Z_{n1n1} & Z_{n1n2} & Z_{n1n3} \\
Z_{n2a} & Z_{n2b} & Z_{n2c} & Z_{n2n1} & Z_{n2n2} & Z_{n2n3} \\
Z_{nma} & Z_{nmb} & Z_{nmc} & Z_{nmn1} & Z_{nmn2} & Z_{nmn3}
\end{bmatrix}
\]

In a typical power system network, each bus is connected by a few nearby bus, which cause many off-diagonal elements to be zero. The \( Y \) matrix is indeed a sparse matrix. If there is no connection between two adjacent nodes, a zero is left in the matrix.
That in the partitioned form might also result like:

\[
\begin{bmatrix}
Z_{\text{primitive}}
\end{bmatrix}
= 
\begin{bmatrix}
Z_{ij} & Z_{in} \\
Z_{nj} & Z_{nn}
\end{bmatrix}
\]

5.4 System Complex Admittance Matrix

The entire matrix is built by placing branch element sub-matrices into the system through a topology matrix. In the same way, earth connections may be defined by adding self-admittances to the neutral conductor at the relative bus. It is generally the branch matrix to be constructed firstly and then this primary block is expanded adding the ones regarding the connection admittances and the loads or generators admittances.

Loads are usually expressed as a series admittance, following the last segment obtained from the branch admittances’ representation. If any constant-power load is present, an iterative method would be necessary to evaluate the equivalent current injection depending on the nodal voltage.

Generators are represented by a complex vector composed of two shares: one made of voltages’ magnitude and phase, the other made of unknown elements.

Considering now the network under analysis, there will be a matrix representing the 4-wire branches that link each pillar to each others to which are added the parts of the consumers’ connections’ admittances and the loads and generators’ shunt admittances.
So in this case, a 188 x 188 matrix will be obtained, composed by (4 Lines x 10 Pillars) + (2 Lines x 74 consumers).

5.5 Network’s equations

Branch elements are activated by the slack node, which is not included in the network’s matrix representation. Depending on the slack bus voltage at a N node of branch admittance matrix, the components of the shunt admittance matrix are subjected to an injection (or absorption) of complex power. In this way, the network equation can be considered in the linear form:

\[
\begin{bmatrix}
I
\end{bmatrix} = \begin{bmatrix}
Y
\end{bmatrix} \begin{bmatrix}
E
\end{bmatrix}
\]

\[
I = \text{nodal injected currents}
\]

\[
E = \text{nodal applied voltages}
\]
Where:

\[ Y = Y_N + Y_{SH} \]

So now the matrix describing the network can be re-written as:

\[
\begin{bmatrix}
   i_a \\
   0 \\
   ... \\
   0
\end{bmatrix}
= 
\begin{bmatrix}
   Y_{GG} & Y_{GL} \\
   Y_{LG} & Y_{LL}
\end{bmatrix}
\begin{bmatrix}
   u_a \\
   ... \\
   u_m
\end{bmatrix}
\]

That also means:

\[ i_G = Y_{GG} \ast u_G + Y_{GL} \ast u_L \]

\[ 0 = Y_{GL} \ast u_G + Y_{LL} \ast u_L \]

That leads to:

\[ u_L = -Y_{LL}^{-1} \ast Y_{LG} \ast u_G \]

and consequently to:

\[ i_G = Y_{GG} + Y_{GL} \ast (-Y_{LL}^{-1} \ast Y_{LG}) \ast u_G \]
The above matrix can be again divided according to the last current equation derived. In this way the system can be seen as formed as:

\[
\begin{bmatrix}
i_a \\
i_x
\end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} u_{ar} \\
u_x
\end{bmatrix}
\]

Since \([ I]\) stands for all the currents that are injected and that \(i_x\) represents only the loads’ currents, \(i_x = 0\):

\[
0 = C \cdot u_{ar} + D \cdot u_x
\]

\[
u_x = - D^{-1} \cdot C \cdot u_{ar}
\]

The system supply current can be described with the equation:

\[
i_a = A \cdot u_{ar} + B \cdot (D^{-1} \cdot C \cdot u_{ar})
\]

This procedure can be applied at any section of the net, to evaluate voltage and current at each segment.

The power flow problem can be iteratively solved through the application of a certain voltage at the slack node, in association with a network admittance matrix representation. The solution can be obtained by setting a flat start to the shunt elements and each network bus. Successive values of system voltage are acquired from every iteration by varying the shunt admittances of both generator and load buses in order to satisfy the required voltage and power values.
5.6 CAM Algorithm structure

5.6.1 Main

The algorithm is structured in terms of four components:

1. a program to facilitate the derivations of the 4-wire (backbone) system y-matrix;

2. a program that facilitates the inclusion of the 2-wire (consumer connections);

3. the pillar earth electrode resistance is compiled as a matrix which is added to the system y-matrix; the consumer earth electrode representation is included when the consumer loads are referenced;

4. the load flow solution is derived in two stages: the consumer load is called, the load flow is implemented.

5.6.2 4-wire and 2-wire admittance matrixes

There are two programs that contain functions to read, CVS files (4wirebuses.csv and 2wirebuses.csv), that recall definitions to describe the lines as considered from bus (fb), to-bus (tb), the line configuration (that defines the type of 4-wire cable) and the line length.

The 4-wire Y-matrix is derived in terms of the fb/tb parameters in terms of the off diagonal and then diagonals.

After gaining the cognisance of the consumer connections to the 4-wire backbone, the Y-matrix system can be extended to represent the single-phase consumer line connections individually.
5.6.3 Earth Elect

This programme defines, in terms of the network description parameters, the connections of the pillar earth electrodes, which means, each pillar accommodates an earth electrode. The programme also defines the consumer earth electrode resistance.

5.6.4 2-Wire Y system

This program has four principal aspects:

1) one subprogram defines the flat voltage configuration;

2) the voltage difference tolerance, iteration count and the maximum number of iterations allowed is defined;

3) a function is used to redefine the consumer load into an admittance proportional to the derived consumer voltage;

4) this function load into a 2x2 construct which can be included in the system Y-matrix.

5.6.5 Results

This file derives a data file which presents the voltage calculated across the network, in terms of the 4-wire backbone and single-phase consumer connections. The idea promoted at the DIT was to gain a quicker and more generalizable procedure.
Thanks to this method a solution can be found, managing to include the Carson-Clem’s equations.
Chapter 6

Conclusions

6.1 The Actual Comparison

The two different programs described propose not so different results when earth electrodes are not included in the b-f iteration. It is anyway possible to guess which is the more reliable one, since the CAM methodology has shown from the beginning clearer methodology and development. The same principle has also been re-used by our Department at the University of Padova to elaborate another program with the same purposes.

In order to better analyse the peculiarities of these approaches, a comparison regarding the results obtained from each of them are highlighted. It must be repeated though that the results here reported from the b-f refer to the absence of the earth connection. In fact including this connection the results are unavailable.

The results obtained by the b-f method with Ciric’s approach show reliability because of the specific type of network under consideration. Kron simplifications, that had being used, can be thought as representative of the Irish network structure that includes the use of the multigrounded neutral. This is the reason why once including Carson - Clem’s formulas the program will not reach a solution, though in this way it would provide a more valid approach since it could be applied to any type of network.

Since, as far as I am concerned, the program created in Ireland has a coherent and reliable structure, the problem of its in-adaptability must be looked up somewhere in its principles, as so, in the application of Carson’s formulas. So I would say that the similarities of the results of the different approaches depend on the use of Kron’s
reduction. This approach applied to that type of network representation affects the vision of the network itself, making it almost irrelevant the values of the grounding impedances of the neutral with regard to the loads and the pillars, thus giving much weight to the grounding in the substation A.

Anyway since with the CAM another way for reaching the same results had been studied and created, resulting more reliable and adaptive, the complex admittance method with current injection correction is the one suggested.

Smaller networks of 5 and 7 pillars are proposed for comparisons. In order to have compatible results, the earth connection for the backward-forward were not involved.
RESULTS

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**Table 1**

| E | 119.8965 | 337.3452 | 527.4586 | 120.4586 | 334.5668 | 0.1708 | 336.0945 |
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| C | 119.9245 | 337.8924 | 527.8924 | 120.8924 | 334.8924 | 0.00822 | 338.2020 |
| B | 119.9243 | 337.8924 | 527.8924 | 120.8924 | 334.8924 | 0.00823 | 338.2023 |
| A | 119.8668 | 337.8668 | 527.8668 | 120.8668 | 334.8668 | 0.00866 | 338.8686 |

**Pillar Voltages**

*Reduction to Source + 5 MINI-Pillars*
REDUCTION TO SOURCE + 5 MINI-PILLARS - CAM METHOD

RESULTS

Neutral Voltage

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**Pillar Voltages**

**Reduction To Source + 5 Mini-Pillars - CWM Method**
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</tr>
<tr>
<td>UU</td>
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<td>1511.7066</td>
</tr>
<tr>
<td>VV</td>
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<td>1553.8809</td>
<td>1553.8809</td>
<td>1553.8809</td>
</tr>
<tr>
<td>WW</td>
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<td>1596.0552</td>
<td>1596.0552</td>
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</tr>
<tr>
<td>XX</td>
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<tr>
<td>YY</td>
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<td>1680.4038</td>
<td>1680.4038</td>
<td>1680.4038</td>
</tr>
<tr>
<td>ZZ</td>
<td>1722.5781</td>
<td>1722.5781</td>
<td>1722.5781</td>
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</tr>
</tbody>
</table>

**PILLAR VOLTAGES**

**Reduction to Source ± MINI-PILLARS B - F Method**
<table>
<thead>
<tr>
<th>Bus</th>
<th>Line 1</th>
<th>Line 2</th>
<th>Line 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.05</td>
<td>0.86</td>
<td>0.86</td>
</tr>
</tbody>
</table>

**Pillar Voltages**

Reduction to Source + 7 Mini-Pillars CAM Method
Appendix A

Backward-forward Method

Model_Entire_Network

clear all
close all
clc

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

% SYSTEM PARAMETERS
V_line = 415;
Sys_MVA_base = 1e+6;
z_base = V_line^2/(Sys_MVA_base);

% Earth Electrode(s)
R_Pillar_Electrode=1/z_base;
R_cons_electrode=10/z_base;

System_Z;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

%-------------------------------------------------------------------
------%
%Initialisation
%-------------------------------------------------------------------
------%

Initial_Voltage % Assigning the flat start to all system busses
Customer_Load_No_EarthElectrode % Deriving the consumer load based o the flat start
Branch_Current % Evaluating the branch currents (incorporaig the pillar current

Diff=zeros(4,1); %Using X3 as the means to ascertain voltage profile

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

%-------------------------------------------------------------------
------%
%Load Flow Loop - Solution without Consumer Earth Electrodes
%-------------------------------------------------------------------
------%

for i=1:4000

72
V_Diff=V_5;   i

if max(abs(V_Diff-V_5))<0.00001
    break
end

Customer_Load_No_EarthElectrode

Branch_Current

(iincorporaig the pillar current
V_5
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%-------------------------------------------------------------------
------%
%Load Flow Loop - Addition of Consumer Earth Electrodes
%-------------------------------------------------------------------
------%
for ii=1:4000
    V_Diff=V_5;   ii
    Voltage_Update
    if max(abs(V_Diff-V_5))<0.00001
        break
    end
    Customer_Load_With_EarthElectrode
    Branch_Current
V_5
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

A=[V2_unbalance V3_unbalance V4_unbalance V5_unbalance V6_unbalance
V7_unbalance V8_unbalance V9_unbalance V10_unbalance]*100;
A'

RESULTS

Initial_Voltage

V_ground=0;
V_neutral=0;

V=[(1+0i);(1*exp(-j*120*pi/180));(1*exp(-j*240*pi/180));V_neutral];

V_1=V;
V_2=V; V_120=V; V_26=V; V_59=V; V_75=V; V_63=V; V_46=V;
V_3=V; V_27=V; V_28=V; V_32=V; V_25=V; V_33=V; V_35=V; V_36=V;
Customer_Load_With_EarthElectrode

% Node (Customer Load) Currents
%MINT-PILLAR A1
bus=120; [S_120 IL_120]=Sys_Load(bus,V_120);
bus=26; [S_26 IL_26]=Sys_Load(bus,V_26);
bus=59; [S_59 IL_59]=Sys_Load(bus,V_59);
bus=75; [S_75 IL_75]=Sys_Load(bus,V_75);
bus=63; [S_63 IL_63]=Sys_Load(bus,V_63);
bus=46; [S_46 IL_46]=Sys_Load(bus,V_46);
%MINT-PILLAR A2
bus=27; [S_27 IL_27]=Sys_Load(bus,V_27);
bus=28; [S_28 IL_28]=Sys_Load(bus,V_28);
bus=32; [S_32 IL_32]=Sys_Load(bus,V_32);
bus=25; [S_25 IL_25]=Sys_Load(bus,V_25);
bus=33; [S_33 IL_33]=Sys_Load(bus,V_33);
bus=35; [S_35 IL_35]=Sys_Load(bus,V_35);
bus=36; [S_36 IL_36]=Sys_Load(bus,V_36);
%MINT-PILLAR A3
bus=37; [S_37 IL_37]=Sys_Load(bus,V_37);
bus=34; [S_34 IL_34]=Sys_Load(bus,V_34);
bus=39; [S_39 IL_39]=Sys_Load(bus,V_39);
bus=48; [S_48 IL_48]=Sys_Load(bus,V_48);
bus=41; [S_41 IL_41]=Sys_Load(bus,V_41);
bus=49; [S_49 IL_49]=Sys_Load(bus,V_49);
bus=72; [S_72 IL_72]=Sys_Load(bus,V_72);
bus=67; [S_67 IL_67]=Sys_Load(bus,V_67);
bus=64; [S_64 IL_64]=Sys_Load(bus,V_64);
%MINT-PILLAR A4
bus=47; [S_47 IL_47]=Sys_Load(bus,V_47);
bus=38; [S_38 IL_38]=Sys_Load(bus,V_38);
bus=61; [S_61 IL_61]=Sys_Load(bus,V_61);
bus=50; [S_50 IL_50]=Sys_Load(bus,V_50);
bus=56; [S_56 IL_56]=Sys_Load(bus,V_56);
bus=40; [S_40 IL_40]=Sys_Load(bus,V_40);
bus=44; [S_44 IL_44]=Sys_Load(bus,V_44);
%MINT-PILLAR A5
bus=62; [S_62 IL_62]=Sys_Load(bus,V_62);
bus=45; [S_45 IL_45]=Sys_Load(bus,V_45);
bus=58; [S_58 IL_58]=Sys_Load(bus,V_58);
bus=57; [S_57 IL_57]=Sys_Load(bus,V_57);
bus=53; [S_53 IL_53]=Sys_Load(bus,V_53);
bus=60; [S_60 IL_60]=Sys_Load(bus,V_60);
bus=43; [S_43 IL_43]=Sys_Load(bus,V_43);
bus=52; [S_52 IL_52]=Sys_Load(bus,V_52);
%MINT-PILLAR A5a
bus=71; [S_71 IL_71]=Sys_Load(bus,V_71);
bus=79; [S_79 IL_79]=Sys_Load(bus,V_79);
bus=77; [S_77 IL_77]=Sys_Load(bus,V_77);
bus=76; [S_76 IL_76]=Sys_Load(bus,V_76);
bus=73;  \([S_{73} IL_{73}]=\text{Sys\_Load}(bus,V_{73})\);
bus=68;  \([S_{68} IL_{68}]=\text{Sys\_Load}(bus,V_{68})\);
bus=69;  \([S_{69} IL_{69}]=\text{Sys\_Load}(bus,V_{69})\);
bus=78;  \([S_{78} IL_{78}]=\text{Sys\_Load}(bus,V_{78})\);
bus=70;  \([S_{70} IL_{70}]=\text{Sys\_Load}(bus,V_{70})\);

%MINI-PILLAR A6
bus=55;  \([S_{55} IL_{55}]=\text{Sys\_Load}(bus,V_{55})\);
bus=74;  \([S_{74} IL_{74}]=\text{Sys\_Load}(bus,V_{74})\);
bus=54;  \([S_{54} IL_{54}]=\text{Sys\_Load}(bus,V_{54})\);
bus=42;  \([S_{42} IL_{42}]=\text{Sys\_Load}(bus,V_{42})\);
bus=51;  \([S_{51} IL_{51}]=\text{Sys\_Load}(bus,V_{51})\);
bus=65;  \([S_{65} IL_{65}]=\text{Sys\_Load}(bus,V_{65})\);
bus=66;  \([S_{66} IL_{66}]=\text{Sys\_Load}(bus,V_{66})\);
bus=1326;  \([S_{1326} IL_{1326}]=\text{Sys\_Load}(bus,V_{1326})\);

%MINI-PILLAR X2
bus=166;  \([S_{166} IL_{166}]=\text{Sys\_Load}(bus,V_{166})\);
bus=165;  \([S_{165} IL_{165}]=\text{Sys\_Load}(bus,V_{165})\);
bus=168;  \([S_{168} IL_{168}]=\text{Sys\_Load}(bus,V_{168})\);
bus=162;  \([S_{162} IL_{162}]=\text{Sys\_Load}(bus,V_{162})\);
bus=167;  \([S_{167} IL_{167}]=\text{Sys\_Load}(bus,V_{167})\);
bus=164;  \([S_{164} IL_{164}]=\text{Sys\_Load}(bus,V_{164})\);
bus=173;  \([S_{173} IL_{173}]=\text{Sys\_Load}(bus,V_{173})\);
bus=169;  \([S_{169} IL_{169}]=\text{Sys\_Load}(bus,V_{169})\);
bus=156;  \([S_{156} IL_{156}]=\text{Sys\_Load}(bus,V_{156})\);
bus=171;  \([S_{171} IL_{171}]=\text{Sys\_Load}(bus,V_{171})\);

%MINI-PILLAR X3
bus=159;  \([S_{159} IL_{159}]=\text{Sys\_Load}(bus,V_{159})\);
bus=178;  \([S_{178} IL_{178}]=\text{Sys\_Load}(bus,V_{178})\);
bus=177;  \([S_{177} IL_{177}]=\text{Sys\_Load}(bus,V_{177})\);
bus=179;  \([S_{179} IL_{179}]=\text{Sys\_Load}(bus,V_{179})\);
bus=176;  \([S_{176} IL_{176}]=\text{Sys\_Load}(bus,V_{176})\);
bus=163;  \([S_{163} IL_{163}]=\text{Sys\_Load}(bus,V_{163})\);
bus=161;  \([S_{161} IL_{161}]=\text{Sys\_Load}(bus,V_{161})\);
bus=172;  \([S_{172} IL_{172}]=\text{Sys\_Load}(bus,V_{172})\);
bus=175;  \([S_{175} IL_{175}]=\text{Sys\_Load}(bus,V_{175})\);
bus=174;  \([S_{174} IL_{174}]=\text{Sys\_Load}(bus,V_{174})\);

Sys\_Load

\textbf{function}  \([Sabc, Iabcng]=\text{Sys\_Load}(bus, V)\)
bus;
\textbf{Sys\_MVA\_base} = 1e+6;
load \texttt{Load.txt}\n\texttt{Load(:,[2:7])=(Load(:,[2:7])*1000)/(Sys\_MVA\_base/3)};
\%Load(:,1) = = Bus\ ID
\%Load(:,2) = = P\_L1
\%Load(:,3) = = Q\_L1
\%Load(:,4) = = P\_L2
\%Load(:,5) = = Q\_L2
\%Load(:,6) = = P\_L3
\%Load(:,7) = = Q\_L3
row=Load(:,1);
for i=1:length(Load(:,1));
    loc(i)=row(i)-bus;
end
pos=find(loc==0);
row_ID=pos(1,1);

Sabc=[Load(row_ID,2)+j*Load(row_ID,3) Load(row_ID,4)+j*Load(row_ID,5) Load(row_ID,6)+j*Load(row_ID,7)];
% V(4)
% pause
Ia=conj(Sabc(1))./(conj(V(1)-V(4)));% pause
Ib=conj(Sabc(2))./(conj(V(2)-V(4)));% pause
Ic=conj(Sabc(3))./(conj(V(3)-V(4)));% pause
Rcons=.15;

In=-(Ia+Ib+Ic)+(V(4))/Rcons;% Ig

%pause
Iabcng=[Ia;Ib;Ic;In];%pause

Branch_Current

%Pillar X3
Pillar=9;
I_174=IL_174;I_175=IL_175;I_172=IL_172;I_161=IL_161;I_163=IL_163;I_176=IL_176;I_179=IL_179;I_177=IL_177;I_178=IL_178;I_159=IL_159;

I_10=I_174+I_175+I_172+I_161+I_163+I_176+I_179+I_177+I_178+I_159;
% pause
I=I_10; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar,I,V_10,Rg_pillar);
I_10=I_Pillar;

%Pillar X2
Pillar=8;
I_171=IL_171;I_156=IL_156;I_169=IL_169;I_173=IL_173;I_164=IL_164;I_167=IL_167;I_162=IL_162;I_168=IL_168;I_165=IL_165;I_166=IL_166;

I_9=I_171+I_156+I_169+I_173+I_164+I_167+I_162+I_168+I_165+I_166+I_10;
I=I_9; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar,I,V_9,Rg_pillar);
I_9=I_Pillar;
%Pillar A6
Pillar=7;
I_1326=IL_1326;I_66=IL_66;I_65=IL_65;I_51=IL_51;I_42=IL_42;I_54=IL_54;I_74=IL_74;I_55=IL_55;
I_8=I_1326+I_66+I_65+I_51+I_42+I_54+I_74+I_55+I_9;
I=I_8; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar,I,V_8,Rg_pillar);
I_8=I_Pillar;

%Pillar A5a
Pillar=6;
I_70=IL_70;I_78=IL_78;I_69=IL_69;I_68=IL_68;I_73=IL_73;I_76=IL_76;I_77=IL_77;I_79=IL_79;I_71=IL_71;
I_7=I_70+I_78+I_69+I_68+I_73+I_76+I_77+I_79+I_71;
I=I_7; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar,I,V_7,Rg_pillar);
I_7=I_Pillar;

%Pillar A5
Pillar=5;
I_62=IL_62;I_45=IL_45;I_58=IL_58;I_57=IL_57;I_53=IL_53;I_60=IL_60;I_43=IL_43;I_52=IL_52;
I_6=I_62+I_45+I_58+I_57+I_53+I_60+I_43+I_52+I_7+I_8;
I=I_6; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar,I,V_6,Rg_pillar);
I_6=I_Pillar;

%Pillar A4
Pillar=4;
I_47=IL_47;I_38=IL_38;I_61=IL_61;I_50=IL_50;I_56=IL_56;I_40=IL_40;I_44=IL_44;
I_5=I_47+I_38+I_61+I_50+I_56+I_40+I_44+I_6;
I=I_5; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar,I,V_5,Rg_pillar);
I_5=I_Pillar;

%Pillar A3
Pillar=3;
I_64=IL_64;I_67=IL_67;I_72=IL_72;I_49=IL_49;I_41=IL_41;I_48=IL_48;I_39=IL_39;I_34=IL_34;I_37=IL_37;
I_4=I_64+I_67+I_72+I_49+I_41+I_48+I_39+I_34+I_37+I_5;
I=I_4; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar,I,V_4,Rg_pillar);
I_4=I_Pillar;

%Pillar A2
Pillar=2;
I_36=IL_36;I_35=IL_35;I_33=IL_33;I_25=IL_25;I_32=IL_32;I_27=IL_27;I_28=IL_28;
I_3=I_36+I_35+I_33+I_25+I_32+I_27+I_28+I_4;
I=I_3; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar,I,V_3,Rg_pillar);
I_3=I_Pillar;

%Pillar A1
Pillar=1;
I_46=IL_46;I_63=IL_63;I_75=IL_75;I_59=IL_59;I_26=IL_26;I_120=IL_120;
I_2=I_46+I_63+I_75+I_59+I_26+I_120+I_3;
I=I_2; Rg_pillar=R_Pillar_Electrode;
[I_Pillar]= Pillar_Current(Pillar, I, V_2, Rg_pillar);
I_2=I_Pillar;

% SUBSTATION - A
I_1=I_2;

Pillar_Current

function [I_Pillar]= Pillar_Current(Pillar, I, V, Rg_pillar)
Pillar;

%          In=I(4)-(V(4)-V(5))/Rg_pillar; % Modified KS (020912)
%          Ig=I(5)+(V(4)-V(5))/Rg_pillar; % Modified KS (020912)
In=I(4)+(V(4))/Rg_pillar; % MC
I_Pillar=[I(1);I(2); I(3); In;];
%pause
%sum(I_Pillar);
%pause
end

Voltage_Update

%-------- SOURCE ---------%
ZE=0.00001;
IE=-(I_1(1)+I_1(2)+I_1(3)+I_1(4));
V_1(4)=V_1(4)+IE*ZE;
V_1=[(1+0i)+V_1(4);(1*exp(-j*120*pi/180))+V_1(4);(1*exp(-j*240*pi/180))+V_1(4);V_1(4)]; % in pu

%-------- Mini-pillar A1 ---------%
V_2=(V_1-(z_A_A1*I_2));
[V2_unbalance]=Voltage_Unbalance(V_2);
V_120=V_2-(z_A1_C120*I_120);
V_26=V_2-(z_A1_C26*I_26);
V_59=V_2-(z_A1_C59*I_59);
V_75=V_2-(z_A1_C75*I_75);
V_63=V_2-(z_A1_C63*I_63);
V_46=V_2-(z_A1_C46*I_46);
%-------- Mini-pillar A2 ---------%
V_3=(V_2-(z_A1_A2*I_3));
Diff;
% pause
[\text{V3\_unbalance}] = \text{Voltage\_Unbalance}(V_3);
  V_{36} = V_3 - (z_{A2\_C36} \cdot I_{36});
  V_{35} = V_3 - (z_{A2\_C35} \cdot I_{35});
  V_{33} = V_3 - (z_{A2\_C33} \cdot I_{33});
  V_{25} = V_3 - (z_{A2\_C25} \cdot I_{25});
  V_{32} = V_3 - (z_{A2\_C32} \cdot I_{32});
  V_{27} = V_3 - (z_{A2\_C27} \cdot I_{27});
  V_{28} = V_3 - (z_{A2\_C28} \cdot I_{28});
%-------- Mini-pillar A3 ---------%
  V_4 = (V_3 - (z_{A2\_A3} \cdot I_4));
  [\text{V4\_unbalance}] = \text{Voltage\_Unbalance}(V_4);
  V_{41} = V_4 - (z_{A3\_C41} \cdot I_{41});
  V_{48} = V_4 - (z_{A3\_C48} \cdot I_{48});
  V_{39} = V_4 - (z_{A3\_C39} \cdot I_{39});
  V_{34} = V_4 - (z_{A3\_C34} \cdot I_{34});
  V_{37} = V_4 - (z_{A3\_C37} \cdot I_{37});
  V_{64} = V_4 - (z_{A3\_C64} \cdot I_{64});
  V_{67} = V_4 - (z_{A3\_C67} \cdot I_{67});
  V_{72} = V_4 - (z_{A3\_C72} \cdot I_{72});
  V_{49} = V_4 - (z_{A3\_C49} \cdot I_{49});
%-------- Mini-pillar A4 ---------%
  V_5 = (V_4 - (z_{A3\_A4} \cdot I_5));
  [\text{V5\_unbalance}] = \text{Voltage\_Unbalance}(V_5);
  V_{47} = V_5 - (z_{A4\_C47} \cdot I_{47});
  V_{38} = V_5 - (z_{A4\_C38} \cdot I_{38});
  V_{61} = V_5 - (z_{A4\_C61} \cdot I_{61});
  V_{50} = V_5 - (z_{A4\_C50} \cdot I_{50});
  V_{56} = V_5 - (z_{A4\_C56} \cdot I_{56});
  V_{40} = V_5 - (z_{A4\_C40} \cdot I_{40});
  V_{44} = V_5 - (z_{A4\_C44} \cdot I_{44});
%-------- Mini-pillar A5 ---------%
  V_6 = (V_5 - (z_{A4\_A5} \cdot I_6));
  [\text{V6\_unbalance}] = \text{Voltage\_Unbalance}(V_6);
  V_{62} = V_6 - (z_{A5\_C62} \cdot I_{62});
  V_{45} = V_6 - (z_{A5\_C45} \cdot I_{45});
  V_{58} = V_6 - (z_{A5\_C58} \cdot I_{58});
  V_{57} = V_6 - (z_{A5\_C57} \cdot I_{57});
  V_{53} = V_6 - (z_{A5\_C53} \cdot I_{53});
  V_{60} = V_6 - (z_{A5\_C60} \cdot I_{60});
  V_{43} = V_6 - (z_{A5\_C43} \cdot I_{43});
  V_{52} = V_6 - (z_{A5\_C52} \cdot I_{52});
%-------- Mini-pillar A5a ---------%
  V_7 = (V_6 - (z_{A5\_A5a} \cdot I_7));
  [\text{V7\_unbalance}] = \text{Voltage\_Unbalance}(V_7);
  V_{71} = V_7 - (z_{A5a\_C71} \cdot I_{71});
  V_{79} = V_7 - (z_{A5a\_C79} \cdot I_{79});
  V_{77} = V_7 - (z_{A5a\_C77} \cdot I_{77});
V_76=V_7-(z_A5a_C76*I_76);
V_73=V_7-(z_A5a_C73*I_73);
V_68=V_7-(z_A5a_C68*I_68);
V_69=V_7-(z_A5a_C69*I_69);
V_78=V_7-(z_A5a_C78*I_78);
V_70=V_7-(z_A5a_C70*I_70);

%-------- Mini-pillar A6 ---------%
V_8=(V_6-(z_A5_A6*I_8));
[V8_unbalance]=Voltage_Unbalance(V_8);
V_1326=V_8-(z_A6_C1326*I_1326);
V_66=V_8-(z_A6_C66*I_66);
V_65=V_8-(z_A6_C65*I_65);
V_51=V_8-(z_A6_C51*I_51);
V_42=V_8-(z_A6_C42*I_42);
V_54=V_8-(z_A6_C54*I_54);
V_74=V_8-(z_A6_C74*I_74);
V_55=V_8-(z_A6_C55*I_55);

%-------- Mini-pillar X2 ---------%
V_9=(V_8-(z_A6_X2*I_9));
[V9_unbalance]=Voltage_Unbalance(V_9);
V_164=V_9-(z_X2_C164*I_164);
V_173=V_9-(z_X2_C173*I_173);
V_169=V_9-(z_X2_C169*I_169);
V_156=V_9-(z_X2_C156*I_156);
V_171=V_9-(z_X2_C171*I_171);
V_166=V_9-(z_X2_C166*I_166);
V_165=V_9-(z_X2_C165*I_165);
V_168=V_9-(z_X2_C168*I_168);
V_162=V_9-(z_X2_C162*I_162);
V_167=V_9-(z_X2_C167*I_167);

%-------- Mini-pillar X3 ---------%
V_10=(V_9-(z_X2_X3*I_9));
[V10_unbalance]=Voltage_Unbalance(V_10);
V_163=V_10-(z_X3_C163*I_1326);
V_161=V_10-(z_X3_C161*I_161);
V_172=V_10-(z_X3_C172*I_172);
V_175=V_10-(z_X3_C175*I_175);
V_174=V_10-(z_X3_C174*I_174);
V_159=V_10-(z_X3_C159*I_159);
V_178=V_10-(z_X3_C178*I_178);
V_177=V_10-(z_X3_C177*I_177);
V_179=V_10-(z_X3_C179*I_179);
V_176=V_10-(z_X3_C176*I_176);
Z_Branch

function Zabcng= Z_Branch(c_length, line_config) % needs to be converted to 'per-miles

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%----------------------------- CONSTANTS
---------------------------------%
f=50;
V_line = 415;
Sys_MVA_base = 1e+6;
z_base = V_line^2/(Sys_MVA_base);
rho=100;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

if line_config == 1 % 1 == 70-XLPE [TEST 3-phase Cable (as per Ciric paper)]
f=50;
rho=100;
ri_line=0.569;
Rad_con=0.01025/2;
GMR_line=0.7788*Rad_con;

ri_n=0.569;
GMR_n=0.7788*Rad_con;

%% GEOMETRIC DISTANCES
%%
%Geometric spacing of the f-core cable
a=[0,0];
b=[0.0109,0];
c=[0,-0.0109];
n=[0.0109, -0.0109];

% Cable HeightDepth 1m (Not sure if this is a plausible depth)
ha=2.00-0.00545;  % Heights/Depths based on dimensions derived in 'UG XLPE.doc'
hb=2.00-0.00545;
hc=2.00+0.00545;
hn=2.00+0.00545;

%Conductor Horizontal Distrances

\[
\begin{align*}
\text{dab} &= \frac{10.9}{1000}; \quad \text{dac} = 0; \quad \text{dan} = \frac{10.9}{1000}; \\
\text{dba} &= \text{dab}; \quad \text{dbc} = \frac{10.9}{1000}; \quad \text{dbn} = 0; \\
\text{dca} &= \text{dac}; \quad \text{dcb} = \text{dbc}; \quad \text{dcn} = \frac{10.9}{1000}; \\
\text{dna} &= \text{dan}; \quad \text{dnb} = \text{dbn}; \quad \text{dnc} = \text{dcn};
\end{align*}
\]

%-------------------------------------------------------------------
------%  
\%
\%
\%

%%% DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%%
zaa = (\text{ri}_\text{line} + j \times 4 \times \pi \times (10^{-4}) \times f \times (\log((2 \times \text{ha})/\text{GMR}_\text{line}))); 
\textcolor{red}{zbb = (\text{ri}_\text{line} + j \times 4 \times \pi \times (10^{-4}) \times f \times (\log((2 \times \text{hb})/\text{GMR}_\text{line})));} 
\textcolor{red}{zcc = (\text{ri}_\text{line} + j \times 4 \times \pi \times (10^{-4}) \times f \times (\log((2 \times \text{hc})/\text{GMR}_\text{line})));} 
\textcolor{red}{znn = (\text{ri}_\text{n} + j \times 4 \times \pi \times (10^{-4}) \times f \times (\log((2 \times \text{hn})/\text{GMR}_\text{n})));}
\]

zab = j \times 4 \times \pi \times (10^{-4}) \times f \times \log((\sqrt{dab^2 + (\text{ha} + \text{hb})^2})/(\sqrt{dab^2 + (\text{ha} - \text{hb})^2}));
zac = j \times 4 \times \pi \times (10^{-4}) \times f \times \log((\sqrt{dca^2 + (\text{ha} + \text{hc})^2})/(\sqrt{dca^2 + (\text{ha} - \text{hc})^2}));
zan = j \times 4 \times \pi \times (10^{-4}) \times f \times \log((\sqrt{dna^2 + (\text{ha} + \text{hn})^2})/(\sqrt{dna^2 + (\text{ha} - \text{hn})^2}));
\textcolor{red}{zna = \text{zan};}
\textcolor{red}{zba = \text{zab};}
\textcolor{red}{zbc = j \times 4 \times \pi \times (10^{-4}) \times f \times \log((\sqrt{dcb^2 + (\text{hb} + \text{hc})^2})/(\sqrt{dcb^2 + (\text{hb} - \text{hc})^2}));}
\textcolor{red}{zbn = j \times 4 \times \pi \times (10^{-4}) \times f \times \log((\sqrt{dbn^2 + (\text{hb} + \text{hn})^2})/(\sqrt{dbn^2 + (\text{hb} - \text{hn})^2}));}
\textcolor{red}{znb = \text{zbn};}
\textcolor{red}{zca = \text{zac};}
\textcolor{red}{zcb = \text{zbc};}
\textcolor{red}{zcn = j \times 4 \times \pi \times (10^{-4}) \times f \times \log((\sqrt{dnc^2 + (\text{hc} + \text{hn})^2})/(\sqrt{dnc^2 + (\text{hc} - \text{hn})^2}));}
\textcolor{red}{znc = \text{zcn};}
%
%
-----

%%% Primitive Matrix
%%
\textcolor{red}{Zabcng = [\text{zaa} \ \text{zab} \ \text{zac} \ \text{zan};}
\textcolor{red}{\text{zba} \ \text{zbb} \ \text{zbc} \ \text{zbn};}
\textcolor{red}{\text{zca} \ \text{zcb} \ \text{zcc} \ \text{zcn};}
\textcolor{red}{\text{zna} \ \text{znb} \ \text{znc} \ \text{znn};] \times (c\_length/1000);}

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end

if line_config == 2
% 2 == 185-XLPE [TEST 3-phase Cable (as per Ciric paper)]
f=50;
rho=100;
ri_line=0.212;
Rad_con=0.01573/2;
GMR_line=0.7788*Rad_con;
ri_n=0.212;
GMR_n=0.7788*Rad_con;

%%                             GEOMETRIC DISTANCES
%%
%Geometric spacing of the f-core cable
a=[0,0];
b=[0.01655,0];
c=[0,-0.01655];
n=[0.01655,-0.01655];

% Cable HeightDepth 1m (Not sure if this is a plausible depth)
%---------------------------------------------------------------
ha=2.00-0.008275;  % Heights based on dimensions derived in 'UG
hb=2.00-0.008275;
hc=2.00+0.008275;
hn=2.00+0.008275;

%Conductor Horizontal Distances
%-----------------------------------------------
---

dab=0.01655; dac=0; dan=0.01655;
dba=dab; dbc=0.01655; dbn=0;
dca=dac; dcb=dbc; dcn=0.01655;
dna=dan; dnb=dbn; dnc=dcn;

---

%%                            DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%%
zza=(ri_line+j*4*pi*(10^-4)*f*(log((2*ha)/GMR_line)));
zbz=(ri_line+j*4*pi*(10^-4)*f*(log((2*hb)/GMR_line)));
zc=(ri_line+j*4*pi*(10^-4)*f*(log((2*hc)/GMR_line)));
zn=(ri_n+j*4*pi*(10^-4)*f*(log((2*hn)/GMR_n)));
zab=j*4*pi*(10^-4)*f*log((sqrt(dab^2+(ha+hb)^2))/(sqrt(dab^2+(ha-hb)^2)));  
zac=j*4*pi*(10^-4)*f*log((sqrt(dac^2+(ha+hc)^2))/(sqrt(dac^2+(ha-hc)^2)));  
zan=j*4*pi*(10^-4)*f*log((sqrt(dan^2+(ha+hn)^2))/(sqrt(dan^2+(ha-hn)^2)));  
zna=zan;  
zba=zab;  
zbc=j*4*pi*(10^-4)*f*log((sqrt(dbc^2+(hb+hc)^2))/(sqrt(dbc^2+(hb-hc)^2)));  
zbn=j*4*pi*(10^-4)*f*log((sqrt(dbn^2+(hb+hn)^2))/(sqrt(dbn^2+(hb-hn)^2)));  
znb=zbn;  
zca=zac;  
zcb=zbc;  
zcn=j*4*pi*(10^-4)*f*log((sqrt(dcn^2+(hc+hn)^2))/(sqrt(dcn^2+(hc-hn)^2)));  
znc=zcn;  

%-------------------------------------------------------------------  
%%%                          Primitive Matrix  
%%%-------------------------------------------------------------------  
Zabcng=[zaa zab zac zan; 
        zba zbb zbc zbn; 
        zca zcb zcc zcn; 
        zna znb znc znn]*(c_length/1000);  
end  

if line_config == 3  

%%% 70-NAKBA [TEST 3-phase Cable (as per Ciric paper)]  
f=50;  
rho=100;  
ri_line=0.507;  
Rad_con=0.00986/2;  
GMR_line=0.7788*Rad_con;  
ri_n=0.507;  
GMR_n=0.7788*Rad_con;  

%% GEOMETRIC DISTANCES  
%%  
%Geometric spacing of the f-core cable  
a=[0,0];  
b=[0.0107,0];  
c=[0,-0.0107];  
n=[0.0107,-0.0107];
% Cable Height Depth 1m (Not sure if this is a plausible depth)

% Heights based on dimensions derived in 'UG XLPE.doc'

ha=2.00-0.00535;  % Heights based on dimensions derived in 'UG XLPE.doc'
hb=2.00-0.00535;  % NOTE - 1m depth speculatively chosen
hc=2.00+0.00535;
hn=2.00+0.00535;

% Conductor Horizontal Distances

dab=0.0107; dac=0; dan=0.0107;
dba=dab; dbc=0.0107; dbn=0;
dca=dac; dcb=dbc; dcn=0.0107;
dna=dan; dnb=dbc; dnc=dcn;

%% DEFINING THE PRIMITIVE (partitioned) Z-MATRIX

zaa=(ri_line+j*4*pi*(10^-4)*f*(log((2*ha)/GMR_line)));
zbb=(ri_line+j*4*pi*(10^-4)*f*(log((2*hb)/GMR_line)));
zcc=(ri_line+j*4*pi*(10^-4)*f*(log((2*hc)/GMR_line)));
znn=(ri_n+j*4*pi*(10^-4)*f*(log((2*hn)/GMR_n)));

zab=j*4*pi*(10^-4)*f*log((sqrt(dab^2+(ha+hb)^2))/(sqrt(dab^2+(ha-hb)^2)));
zac=j*4*pi*(10^-4)*f*log((sqrt(dac^2+(ha+hc)^2))/(sqrt(dac^2+(ha-hc)^2)));
zan=j*4*pi*(10^-4)*f*log((sqrt(dan^2+(ha+hn)^2))/(sqrt(dan^2+(ha-hn)^2)));
zna=zan;
zba=zab;
zbc=j*4*pi*(10^-4)*f*log((sqrt(dbc^2+(hb+hc)^2))/(sqrt(dbc^2+(hb-hc)^2)));
zbn=j*4*pi*(10^-4)*f*log((sqrt(dbn^2+(hb+hn)^2))/(sqrt(dbn^2+(hb-hn)^2)));
znb=zbn;
zca=zac;
zcb=zbc;
zcn=j*4*pi*(10^-4)*f*log((sqrt(dcn^2+(hc+hn)^2))/(sqrt(dcn^2+(hc-hn)^2)));
znc=zcn;

%% ---

Practice Matrix

Zabcng=[zaa zob zac zan;
    zba zbb zbc zbn;]
if line_config == 4
% 4 == Service (R-Phase); 25/16 AYCY
[[Modelled] 1-phase Cable (as per Ciric paper)]
f=50;
 rho=100;
 ri_line=1.18;
 rad_con=0.00279;
 GMR_line=0.7788*rad_con; % ==rad_con*(exp(-0.25))

% Neutral conductor (concentric)
 K=28; % No. of strands in concentric neutral
 ri_n=1.12;
 rad_nstrand=0.85;
 GMRnstrand = rad_nstrand*(exp(-0.25)); % in feet
 GMR_n = ((GMRnstrand*K*(rad_con^(K-1)))^(1/K)); % equivalent GMR
 of the concentric neutral in feet

%%                             GEOMETRIC DISTANCES
%%
hi=7;   % Heights chosen arbitrarily - house connections...
hn=7;

%Conductor Horizontal Distances
%---------------------------------------------------------------
---$
din=rad_con;

%---------------------------------------------------------------
---$
%% DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%%
zi=(ri_line+j*4*pi*(10^-4)*f*(log((2*hi)/GMR_line))));
zin=j*4*pi*(10^-4)*f*log((sqrt(din^2+(hi+hn)^2))/(sqrt(din^2+(hi-hn)^2))));
zn=zin;
znn=(ri_n+j*4*pi*(10^-4)*f*(log((2*hn)/GMR_n))));

%---------------------------------------------------------------
---$

% Primitive Matrix
% 0 0 0 zin;
% 0 0 0 0;
% 0 0 0 0;
if line_config == 5  % 4 == Service (S-Phase); 25/16 AYCY
[[Modelled] 1-phase Cable (as per Ciric paper)]
f=50;
rho=100;
ri_line=1.18;
rad_con=0.00279;
GMR_line=0.7788*rad_con;  % ==rad_con*(exp(-0.25))

% Neutral conductor (concentric)
K=28;  % No. of strands in concentric neutral
ri_n=1.12;
rad_nstrand=0.85;
GMRnstrand = rad_nstrand*(exp(-0.25));  % in feet
GMR_n = ((GMRnstrand*K*(rad_con^(K-1)))^(1/K));  % equivalent GMR
of the concentric neutral in feet

%% GEOMETRIC DISTANCES
%%
hi=7;  % Heights chosen arbitrarily - house connections...
hn=7;

%Conductor Horizontal Distances
%------------------------------------------
------
din=rad_con;
%------------------------------------------
------
%% DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%%
zi=(ri_line+j*4*pi*(10^-4)*f*(log((2*hi)/GMR_line)));
zin=j*4*pi*(10^-4)*f*log((sqrt(din^2+(hi+hn)^2))/(sqrt(din^2+(hi-hn)^2)));
zi=zin;
znn=(ri_n+j*4*pi*(10^-4)*f*(log((2*hn)/GMR_n)));

%------------------------------------------
------
%% Primitive Matrix
%%
Zabcng=[ 0 0 0 0;
 0 zii 0 zin;
 0 0 0 0];
0  zni  0  znn;]*(c_length)/1000;

end

% if line_config == 6  % 4 == Service (T-Phase); 25/16 AYCY
% [[Modelled] 1-phase Cable (as per Ciric paper)]
f=50;
rho=100;
ri_line=1.18;
rad_con=0.00279;
GMR_line=0.7788*rad_con;  % ==rad_con*(exp(-0.25))

% Neutral conductor (concentric)
K=28;  % No. of strands in concentric neutral
ri_n=1.12;
rad_nstrand=0.85;
GMRnstrand = rad_nstrand*(exp(-0.25));  % in feet
GMR_n = ((GMRnstrand*K*(rad_con^(K-1)))^(1/K));  % equivalent GMR
of the concentric neutral in feet

%%                             GEOMETRIC DISTANCES
%%
hi=7;  % Heights chosen arbitrarily - house connections...
hn=7;

%din=rad_con;

%Conductor Horizontal Distances
%%---------------------------------------------------------------
%%
%% DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%%
%% zii=(ri_line+j*4*pi*(10^-4)*f*(log((2*hi)/GMR_line)));
%% zin=j*4*pi*(10^-4)*f*log((sqrt(din^2+(hi+hn)^2))/(sqrt(din^2+(hi-hn)^2)));
%% zni=zin;
%% znn=(ri_n+j*4*pi*(10^-4)*f*(log((2*hn)/GMR_n)));
%%
%%---------------------------------------------------------------
%%
% Primitive Matrix
%%
Zabcng=[ 0  0  0  0; 0  0  0  0; 0  0  zii  zin;]
0 0 zni znn;J*(c_length)/1000;
end
Appendix B

Main_CAM

clear all
clc

% Add the path to the following directories:
path(1,1)={'\Calcs\'};
path(2,1)={'\Data\'};
for ni=1:size(path,1)
    addpath([pwd path{ni}]);
end

V_line = 415/sqrt(3);
V_neutral=0;
V_flat=[(1+0i);(1*exp(-j*120*pi/180));(1*exp(-j*240*pi/180));V_neutral]; % in pu

Sys_MVA_base = 1e+6;
z_base = (V_line^2)/(Sys_MVA_base);

Four_Wire_Y_Sys; % Creating the 4-wire Y-bus network structure
Two_Wire_Y_Sys; % Accomodating the additional 2-wire line(s) to % the 4-wire (backbone) Y-matrix

Earth_Elec; % Adding the Earth Electrode Resistance % (Cons. & Pillar)

Sys_LF; % Accomodating the Load System Y-matrix & Sys. LF

% Y(188,188)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Results_out
for ni=1:nbranch_4wire*4-1
    E_matrix(1:4,ni)=V_SYS((1:4)+4*(ni-1),1);
end
exp_results(E_matrix,(1:5),it,1);

Network

%Bus informations loading
function [i, fb, tb, line_config, c_length, Y_Load]=Network
    (Wire_number, V);

if Wire_number==4
    Four_wire_buses_file = '4wirebuses.csv';
    Data=importdata(Four_wire_buses_file);
i=strcmp(Data.colheaders,'i');
    fb=strcmp(Data.colheaders,'From_Bus');
    tb=strcmp(Data.colheaders,'To_Bus');
    line_config=strcmp(Data.colheaders,'Line_Config');
    c_length=strcmp(Data.colheaders,'Length');

    Network.Buses=[% Index From Bus To Bus Line Config Length
        Data.data(:,i) Data.data(:,fb) Data.data(:,tb) Data.data(:,line_config) Data.data(:,c_length)];
i=Data.data(:,i);
    fb=Data.data(:,fb);
    tb=Data.data(:,tb);
    line_config=Data.data(:,line_config);
    c_length=Data.data(:,c_length);
end

clear Data Index From_Bus To_Bus Line_Config Length

if Wire_number==2
    Two_wire_buses_file = '2wirebuses.csv';

end

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Data=importdata(Two_wire_buses_file);
i=strcmp(Data.colheaders,'i');
fb=strcmp(Data.colheaders,'From_Bus');
tb=strcmp(Data.colheaders,'To_Bus');
line_config=strcmp(Data.colheaders,'Line_Config');
c_length=strcmp(Data.colheaders,'Length');

Network.Buses=[%Index From Bus To Bus Line Config Length Actual ID
    Data.data(:,i) Data.data(:,fb) Data.data(:,tb) Data.data(:,line_config) Data.data(:,c_length) ];
i=Data.data(:,i);
fb=Data.data(:,fb);
tb=Data.data(:,tb);
line_config=Data.data(:,line_config);
c_length=Data.data(:,c_length);
end

clear Data Index From_Bus To_Bus Line_Config Length

load_conversion

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%                      Load_Conversion
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [ Y_Load,load_pwr ] = Load_Conversion( ii, V,Sys_MVA_base )
%Load_Conversion converts (for each load) the value in kW into impedance
%and then admittance

Load_file ='Load.csv';
Data=importdata(Load_file);
i=strcmp(Data.colheaders,'i');
Load=strcmp(Data.colheaders,'Load');

Sys_Load=[%Index Load
    Data.data(:,i) Data.data(:,Load) ];
i=Data.data(:,i);
Net_Load=Data.data(:,Load);
P_Load=(Net_Load(ii)*1000)/(Sys_MVA_base);

if P_Load>0
    Z_Load=(abs(V((2*ii)-1)-V(2*ii))^2)/conj(P_Load);  % V_ph-V_n
end
Y_Load=inv(Z_Load);

I=Y_Load.*(V((2*ii)-1)-V(2*ii));
load_pwr=conj(I).*(V((2*ii)-1)-V(2*ii));

% This section is utilised to validate the derived voltage in terms of the connected consumer load (as contained with the Load.csv file)

else
Y_Load=0;
load_pwr=0;
end
end

y_ij_col

function [ y_ij ] =y_ij_pad_col(z_1,line_config_2wire)
% This function organises the single-phase (2-wire) connection into a 4x2 construct (which will be entered into Y(single-ph (ij)))

y = zeros(4,2);
y_1=inv((z_1));
if line_config_2wire ==4
y(1,1) = y_1(1,1);
y(1,2) = y_1(1,2);
y(4,1) = y_1(2,1);
y(4,2) = y_1(2,2);
end
if line_config_2wire ==5
y(2,1) = y_1(1,1);
y(2,2) = y_1(1,2);
y(4,1) = y_1(2,1);
y(4,2) = y_1(2,2);
end
if line_config_2wire ==6
y(3,1) = y_1(1,1);
y(3,2)=y_1(1,2);
y(4,1)=y_1(2,1);
y(4,2)=y_1(2,2);
end
y_ij=y;
function [ y_ij ] = y_ij_pad_row(z_1,line_config_2wire);
% This function organises the single-phase (2-wire) connection
% into a 4x2 construct (which will be entered into Y(single-ph (ij)))

y = zeros(2,4);
y_1=inv((z_1));
if
line_config_2wire ==4
y(1,1) = y_1(1,1);
y(1,4) = y_1(1,2);
y(2,1) = y_1(2,1);
y(2,4) = y_1(2,2);
end
if
line_config_2wire ==5
y(1,2) = y_1(1,1);
y(1,4) = y_1(1,2);
y(2,2) = y_1(2,1);
y(2,4) = y_1(2,2);
end
if
line_config_2wire ==6
y(1,3) = y_1(1,1);
y(1,4) = y_1(1,2);
y(2,3) = y_1(2,1);
y(2,4) = y_1(2,2);
end
y_ij=y;
end

function [ y_load ] = y_loadconstruct(Y_Load,Rcons)
% This function organises the single-phase (2-wire) load
% connection (Y_Load)
% into a 2x2 construct (which will be added into Y(jj))

if Y_Load ==0
    Rcons=inf;
end
```matlab
y = zeros(2);
y(1,1) = Y_Load;
y(1,2) = -Y_Load;
y(2,1) = -Y_Load;
y(2,2) = (Y_Load); %+ (1/Rcons));

y_load = y;
end

struct_y

function [r1, r2, c1, c2] = Diag_Struct_Y(m);
% This function facilitates diagonal indexing of the 'from-bus' to the
% 'to-bus' in terms of a 4-wire system, i.e. identifies row/coumn
% positions cognisant of the nature of a 4-conductor system

r1 = ((m-1)*4)+1;
r2 = r1+3;
c1 = ((m-1)*4)+1;
c2 = c1+3;
end

Earth_electrode

Rc = 5; % Consumer Electrode (Ohms)
Rcons = Rc/z_base; % Consumer Earth Electrode pu. The

% consumer earth electrode is
% considered
Rp = 1; % Pillar Electrode (Ohms)
r_pillar = 1/(Rp/z_base); % Y-representation of Pillar Electrode
R_Pillar = zeros((nbranch_2wire)*2+(nbranch_4wire+1)*4,...
(nbranch_2wire)*2+(nbranch_4wire+1)*4);

% aa = 4; bb = 4;
% for i = 4:length(R_Pillar)/4;
%     R_Pillar(aa,bb) = r_pillar;
%     aa = aa + 4;
```
% bb=bb+4;
% end
Pillar_R_ref=((nbranch_4wire*4)+4)
Consumer_R_ref=((nbranch_4wire*4)+4)+(nbranch_2wire*2)
%for ni=4:4:40
for ni=4:4:Pillar_R_ref
    R_Pillar(ni,ni)=r_pillar;
end
%for ni=(Pillar_R_ref+2):2:Consumer_R_ref
for ni=(Pillar_R_ref+2):2:Consumer_R_ref
    R_Pillar(ni,ni)=1/Rcons;
end
Y=Y+R_Pillar; % Adding the Pillar Earth Electrode admittance into % the System Y_Matrix

Four_wire_y_sys

Wire_number=4;
[i,fb, tb, line_config, c_length]=Network(Wire_number); % Function call that reads in
% Function call that reads in the csv
% file associated with the 4-
% no. of 4-wire buses...
% and outputs an index (i),
% configuration % (line_config), cable length
% no. of 4-wire branches...
Y = zeros(nbus*4,nbus*4); % Initialise (4-wire) YBus...

% Formation of the Off Diagonal Elements...
for k=1:nbranch_4wire
    [r1, r2, c1, c2]=Non_Diag_Struct_Y(fb, tb, k);
    Y((r1:r2),(c1:c2)) = -inv(Z_Branch(c_length(k), line_config(k))/
% The contents of the 4-wire Y
    % identified by r1-to-r2 and
    % are populated by the
    % inverse of the Z_Branch
    function
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% Formation of Diagonal Elements....
for m = 1:nbus
    for n = 1:nbranch_4wire
        if fb(n) == m
            \[r_1, r_2, c_1, c_2\] = Diag_Struct_Y(m);
            Y((r_1:r_2),(c_1:c_2)) = Y((r_1:r_2),(c_1:c_2)) + inv\((Z_Branch(c_length(n), line_config(n)) / z_base))\);
        elseif tb(n) == m
            \[r_1, r_2, c_1, c_2\] = Diag_Struct_Y(m);
            Y((r_1:r_2),(c_1:c_2)) = Y((r_1:r_2),(c_1:c_2)) + inv\((Z_Branch(c_length(n), line_config(n)) / z_base))\);
        end
    end
end

Two_wire_y_sys

Wire_number=2;
[i, fb, tb, line_config, c_length] = Network(Wire_number);
nbranch_2wire = length(fb); % no. of branches wrt 2-wire connections ..

for k = 1:nbranch_2wire
    y = i(k);
    % As the load is considered as part of a csv file,
    % an index is required to line the particular
    % node/bus to the data file used for everything
else

%%%%%--- Adding the additional 2-wire line to the 4-wire Y-matrix ---%%%%%

%%%%%--- Creating \((Y(4-wire)+Y(single-ph(ii)))\) -----%

z_l = (Z_Branch(c_length(k), line_config(k)) / z_base);
    % Acquiring z-matrix of the 2-wire connection

end
y_{ii}=y_{2to4wire}(z_1,line_config(k));  % Desconstructing the impedance representation
fit into % into a 2-wire y matrix so that it can
y_{124wire} function % the 4-wire Y-matrix using the

[r1, r2, c1, c2]=Diag_Struct_Y(fb(k));  % Positioning of the 2-wire y matrix
connection % within the 4-wire Y-matrix (Y(4-wire))
Y((r1:r2),(c1:c2))=Y((r1:r2),(c1:c2))+y_{ii};  % 'Adding' the 2-wire y matrix
connection within % the 4-wire Y-matrix (Y(4-wire))

%-------- Matrix Padding (facilitating line connection(s))
------------%
Y(length(Y)+2, length(Y)+2)=0;  % Padding the 4-wire Y-matrix by 2 to
accomodate % the addition of a 2-wire connection (iteratively)

% %---------------------- Y(single-ph(ij)) ---------------------------%
[mm,nn]=size(Y);

y_{ij\_col}= y_{ij\_pad\_col}(z_1,line_config(k));  % y_{ij} is facilitated by the y_{ij\_pad}
function % which re-structures the 2-wire y-matrix
to % align with the y_{ij} structure
Y((r1:r2),(nn-1:nn))=-y_{ij\_col};  % Positioning of the 2-wire y matrix
within the % 4-wire construct

%---------------------- Y(single-ph(ji)) ---------------------------% 

y_{ij\_row}= y_{ij\_pad\_row}(z_1,line_config(k));  % y_{ji} is facilitated by the y_{ij\_pad}
function % which re-structures the 2-wire y-
matrix to % align with the y_{ji} structure
Y((mm-1:mm),(c1:c2))=-y_{ij\_row};

%---------------------- Y(single-ph(jj)) ---------------------------% 

Y((mm-1:mm),(nn-1:nn))=inv(z_1);  %

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% Positioning of the 2-wire $y$ matrix

(Y_{jj}) \text{ in terms of a 4-wire construct}

ey_{\text{2to4_wi}r}e

function [ y_{ii} ] = y_{\text{2to4wire}}(z_1, \text{line_config}_2wire)
% This function organises the single-phase (2-wire) connection $Y_{\text{single-ph(ii)}}$ into a 4x4 construct (which will be added into $Y_{(ii)}$) $[\{Y_{(4-wire)}+Y_{\text{single-ph(ii)}}\}]
\begin{align*}
y &= \text{zeros}(4); \\
y_{1} &= \text{inv}((z_1)); \\
\text{if} & \quad \text{line_config}_2wire == 4 \\
y(1,1) &= y_{1}(1,1); \\
y(1,4) &= y_{1}(1,2); \\
y(4,1) &= y_{1}(2,1); \\
y(4,4) &= y_{1}(2,2); \\
\text{end} \\
\text{if} & \quad \text{line_config}_2wire == 5 \\
y(2,2) &= y_{1}(1,1); \\
y(2,4) &= y_{1}(1,2); \\
y(4,2) &= y_{1}(2,1); \\
y(4,4) &= y_{1}(2,2); \\
\text{end} \\
\text{if} & \quad \text{line_config}_2wire == 6 \\
y(3,3) &= y_{1}(1,1); \\
y(3,4) &= y_{1}(1,2); \\
y(4,3) &= y_{1}(2,1); \\
y(4,4) &= y_{1}(2,2); \\
\text{end}
\end{align*}
\begin{align*}
y_{ii} &= y;
\end{align*}

\text{end}

Non\_diag\_Struct\_Y

function [ r1, r2, c1, c2 ] = Non_Diag_Struct_Y(fb, tb, k);
% This function facilitates non-diagonal indexing of the 'from-bus' to the 'to-bus' in terms of a 4-wire system, i.e. identifies row/column positions cognisant of the nature of a 4-conductor system.
if fb(k) == k
    r1 = ((k-1)*4)+1;
    r2 = r1+3;
else
    if fb(k) ~= k
        r1 = (((k-1)*4)+1)-(k-fb(k))*4;
        r2 = r1+3;
    end
end

if tb(k) == k
    c1 = ((k-1)*4)+1;
    c2 = c1+3;
else
    if tb(k) ~= k
        c1 = (((k-1)*4)+1)-(k-tb(k))*4;
        c2 = c1+3;
    end
end
end

Z_Branch

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%                              Z_Branch
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

function Zabcn = Z_Branch(c_length, line_config)

%%%%%%%%%%%%%%%%%%%%% constants
f = 50;
V_line = 415;
Sys_MVA_base = 1e+6;
% z_base = V_line^2/(Sys_MVA_base);
rho = 100;
De = 659*sqrt(rho/f);
RE = pi^2*f*10^-4;

100
if line_config == 1  % 1 == 70-XLPE [TEST 3-phase Cable (as per Ciric paper)]
f=50;
rho=100;
ri_line=0.569;
Rad_con=0.01025/2;
GMR_line=0.7788*Rad_con;

ri_n=0.569;
GMR_n=0.7788*Rad_con;

%%                             GEOMETRIC DISTANCES
%%                             %Geometric spacing of the f-core cable
a=[0,0];
b=[0.0109,0];
c=[0,-0.0109];
n=[0.0109, -0.0109];

% Cable HeightDepth lm (Not sure if this is a plausible depth)
%-------------------------------------------------------------------
------%
ha=2.00-0.00545;  % Heights/Depths based on dimensions derived in 'UG XLPE.doc'
hb=2.00-0.00545;
hc=2.00+0.00545;
hn=2.00+0.00545;

%Conductor Horizontal Distances
%-------------------------------------------------------------------
------%
dab=10.9/1000; dac=0; dan=10.9/1000;
dba=dab; dbc=10.9/1000; dbn=0;
dca=dac; dcb=dbc; dcn=10.9/1000;
dna=dan; dnb=dnb; dnc=dcn;

%-------------------------------------------------------------------
-------
%%            DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%%
zza=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line)));
zbz=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line)));
zcc=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line)));
zn=(ri_n+RE+j*4*pi*(10^-4)*f*(log(De/GMR_n)));

zab=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dab^2+(ha-hb)^2)));
zac=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dac^2+(ha-hc)^2)));
zan=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dan^2+(ha-hn)^2)));

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zna = zan;
zba = zab;

\[ zbc = \text{RE} + j \times 4 \times \pi \times 10^{-4} \times f \times \log(\text{De}/(\sqrt{\text{dbc}^2 + (\text{hb-hc})^2})); \]

\[ zbn = \text{RE} + j \times 4 \times \pi \times 10^{-4} \times f \times \log(\text{De}/(\sqrt{\text{dbn}^2 + (\text{hb-hn})^2})); \]

\[ \text{zn} = zbn; \]

\[ zcb = zbc; \]

\[ zcn = \text{RE} + j \times 4 \times \pi \times 10^{-4} \times f \times \log(\text{De}/(\sqrt{\text{dcn}^2 + (\text{hc-hn})^2})); \]

\[ \text{zn} = zcn; \]

\% \text{REarth} = (\pi^2 \times f \times 10^{-4}); \ \% \text{[ohm/km]} \text{ unit length resistance of the equivalent earth return conductor.}\n\%
\text{De} = 659 \times \text{sqrt(rho/f)}; \ \% \text{[m]} \text{ distanza/diametro equivalente ritorno correnti nel terreno}\n\%
\%
\%
\%
\w = 2 \times \pi \times f;
\%
\text{zaa} = \text{ri_line} + \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ \text{GMR_line});
\%
\text{zbb} = \text{ri_line} + \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ \text{GMR_line});
\%
\text{zcc} = \text{ri_line} + \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ \text{GMR_line});
\%
\text{znn} = \text{ri_n} + \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ \text{GMR_n});
\%
\%
\%
\%
\text{zab} = \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ \text{dab});
\%
\text{zac} = \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ (\text{sqrt(dac}^2 + (\text{ha-hc})^2)));
\%
\text{zan} = \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ (\text{sqrt(dan}^2 + (\text{ha-hn})^2)));
\%
\%
\%
\%
\text{zaa} = \text{zan};
\%
\text{zba} = \text{zab};
\%
\text{zbc} = \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ (\text{sqrt(dbc}^2 + (\text{hb-hc})^2)));
\%
\text{zbn} = \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ (\text{sqrt(dbn}^2 + (\text{hb-hn})^2)));
\%
\%
\%
\%
\text{zbc} = \text{zbc};
\%
\text{zcb} = \text{zcb};
\%
\text{zcn} = \text{REarth} + j \times w \times 0.46 \times 10^{-3} \times \log10(\text{De}/ (\text{sqrt(dcn}^2 + (\text{hc-hn})^2)));
\%
\%
\%
\%
\text{zn} = \text{zcn};
\%
\text{-----------------------------------------------}
------% Primitive Matrix ------% Zabcn=[zaa zab zac zan; zba zbb zbc zbn; zca zcb zcc zcn; zna znb znc znn]*(c_length/1000);
if line_config == 2  % 2 == 185-XLPE [TEST 3-phase Cable (as per Ciric paper)]
f=50;
rho=100;
ri_line=0.212;
Rad_con=0.01573/2;
GMR_line=0.7788*Rad_con;
ri_n=0.212;
GMR_n=0.7788*Rad_con;

%%                             GEOMETRIC DISTANCES
%%
%Geometric spacing of the f-core cable
a=[0,0];
b=[0.01655,0];
c=[0,-0.01655];
n=[0.01655,-0.01655];

% Cable HeightDepth 1m (Not sure if this is a plausible depth)
%-------------------------------------------------------------------
------
ha=1.00-0.008275;  % Heights based on dimensions derived in 'UG XLPE.doc'
hb=1.00-0.008275;
hc=1.00+0.008275;
hn=1.00+0.008275;

%Conductor Horizontal Distances
%-------------------------------------------------------------------
------

dab=0.01655;  dac=0;dan=0.01655;
dba=dab;  dbc=0.01655;  dbn=0;
dca=dac;  dcb=dbc;  dcn=0.01655;
dna=dan;  dnb=dbn;  dnc=dcn;

%-------------------------------------------------------------------
------
%%            DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%%
zzz=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line))));
zzb=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line))));
zcc=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line))));
znn=(ri_n+RE+j*4*pi*(10^-4)*f*(log(De/GMR_n)));

zab=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dab^2+(ha-hb)^2)));
zac=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dac^2+(ha-hc)^2)));
zan=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dan^2+(ha-hn)^2)));
zna=zan;
zbz=zab;
zbz=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dbc^2+(hb-hc)^2)));
zbn=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dbn^2+(hb-hn)^2)));
%  REarth  = (pi^2*f*10^-4);     % [ohm/km] unit length resistance of 
% the equivalent earth return conductor. 
% De      = 659 *sqrt( rho/f );  % [m] distanza/diametro equivalente 
% ritorno correnti nel terreno 
% w=2*pi*f; 
% zaa = ri_line + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_line); 
% zbb = ri_line + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_line); 
% zcc = ri_line + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_line); 
% znn = ri_n + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_n); 
% 
% zab = REarth +j*w.*0.46*10^-3.*log10(De./ dab); 
% zac = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt((dac^2+(ha-hc)^2))); 
% zan = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt((dan^2+(ha-hn)^2))); 
% zna=zan; 
% zba=zab; 
% zbc = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt((dbc^2+(hb-hc)^2))); 
% zbn = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt((dbn^2+(hb-hn)^2))); 
% znb=zbn; 
% zca=zac; 
% zcb=zbc; 
% zcn = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt((dcn^2+(hc-hn)^2))); 
% znc=zcn; 
% 
%-------------------------------------------------------------------
%------%                                   Primitive Matrix
%------%                                   %%%%%%%%%%%%%%%%%%%%%%%%%%%%

Zabcn=[zaa zab zac zan; 
       zba zbb zbc zbn; 
       zca zcb zcc zcn; 
       zna znb znc znn;]*(c_length/1000);

end

if line_config == 3       % 3 == 70-NAKBA [TEST 3-phase Cable (as per 
                       Ciric paper)] 
f=50; 
rho=100;
ri_line=0.507;
Rad_con=0.00986/2;
GMR_line=0.7788*Rad_con;
ri_n=0.507;
GMR_n=0.7788*Rad_con;

%%                             GEOMETRIC DISTANCES
%%
%Geometric spacing of the f-core cable
a=[0,0];
b=[0.0107,0];
c=[0,-0.0107];
n=[0.0107,-0.0107];

% Cable HeightDepth 1m (Not sure if this is a plausible depth)
%-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-

ha=2.00-0.00535;  % Heights based on dimensions derived in 'UG
hb=2.00-0.00535;  % NOTE - 1m depth speculatively chosen
hc=2.00+0.00535;
hn=2.00+0.00535;

%Conductor Horizontal Distrances
%-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-

%-------------------------------------------
dab=0.0107; dac=0; dan=0.0107;
dba=dab; dbc=0.0107; dbn=0;
dca=dac; dcb=dbc; dcn=0.0107;
dna=dan; dnb=dbn; dnc=dcn;

%-------------------------------------------

% DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%
zaa=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line)));
zbb=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line)));
zcc=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line)));
znn=(ri_n+RE+j*4*pi*(10^-4)*f*(log(De/GMR_n)));

zab=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dab^2+(ha-hb)^2)));
zac=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dac^2+(ha-hc)^2)));
zan=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dan^2+(ha-hn)^2)));
zna=zan;
zba=zab;
zbc=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dbc^2+(hb-hc)^2)));
zbn=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dbn^2+(hb-hn)^2)));
znb=zbn;
zca=zac;
zcb=zbc;
zcn=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(dcn^2+(hc-hn)^2)));

znc=zcn;
% REarth = (pi^2*f*10^-4);       % [ohm/km] unit length resistance of
% the equivalent earth return conductor.
% De      = 659 *sqrt( rho/f );  % [m] distanza/diametro equivalente
% ritorno correnti nel terreno
% w=2*pi*f;
% zaa = ri_line + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_line);
% zbb = ri_line + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_line);
% zcc = ri_line + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_line);
% znn = ri_n + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_n);
% 
% zab = REarth +j*w.*0.46*10^-3.*log10(De./ dab);
% zac = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt(dac^2+(ha-hc)
% ^2)));
% zan = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt(dan^2+(ha-hn)
% ^2)));
% zna=zan;
% zba=zab;
% zbc = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt(dbc^2+(hb-hc)
% ^2)));
% zbn = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt(dbn^2+(hb-hn)
% ^2)));
% znb=zbn;
% zca=zac;
% zcb=zbc;
% zcn = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt(dcn^2+(hc-hn)
% ^2)));
% znc=zcn;
%--------------------------------------------------------------
% Primitive Matrix
%-----------------------------------
Zabcn=[zaa zab zac zan;
        zba zbb zbc zbn;
        zca zcb zcc zcn;
        zna znb znc znn]*(c_length/1000);

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
if line_config == 4   % 4 == Service (R-Phase); 25/16 AYCY
  [[Modelled] 1-phase Cable (as per Ciric paper)]
f=50;
rho=100;
ri_line=1.18;
rad_con=0.00279;
GMR_line=0.7788*rad_con; % ==rad_con*(exp(-0.25))

% Neutral conductor (concentric)
K=28;     % No. of strands in concentric neutral
ri_n=1.12;
rad_nstrand=0.85;
GMRnstrand = rad_nstrand*(exp(-0.25));     % in feet
GMR_n = ((GMRnstrand*K*(rad_con^(K-1)))^(1/K));     % equivalent GMR of the concentric neutral in feet

%%                             GEOMETRIC DISTANCES
%%
hi=4.5;     % Heights chosen arbitrarily – house connections...
hn=4.5;

%Conductor Horizontal Distances
%----------------------------------------------------------------%
din=rad_con;
%----------------------------------------------------------------%

%%            DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%%
%c_length=1000;
zi=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line)));
zin=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(din^2+(hi-hn)^2)));
zni=zin;
znn=(ri_n+RE+j*4*pi*(10^-4)*f*(log(De/GMR_n)));

% REarth = (pi^2*f*10^-4);     % [ohm/km] unit length resistance of the equivalent earth return conductor.
% De = 659 *sqrt( rho/f );     % [m] distanza/diametro equivalente ritorno correnti nel terreno
% w=2*pi*f;
% % zii = ri_line + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_line);
% znn = ri_n + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_n);
% zin = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt(din^2+(hi-hn)^2)));
% zni=zin;
%
%----------------------------------------------------------------%
%%                          Primitive Matrix
%%
Zabcn=[zi  zin; 
      zni  znn;]*(c_length)/1000;

end
if line_config == 5
  % 4 == Service (S-Phase); 25/16 AYCY
  [[Modelled] 1-phase Cable (as per Ciric paper)]
f=50;
rho=100;
ri_line=1.18;
rad_con=0.00279;
GMR_line=0.7788*rad_con; % ==rad_con*(exp(-0.25))

% Neutral conductor (concentric)
K=28; % No. of strands in concentric neutral
ri_n=1.12;
rad_nstrand=0.85;
GMRnstrand = rad_nstrand*(exp(-0.25)); % in feet
GMR_n = ((GMRnstrand*K*(rad_con^(K-1)))^(1/K)); % equivalent GMR of the concentric neutral in feet

%% GEOMETRIC DISTANCES
%%
hi=4.5; % Heights chosen arbitrarily - house connections...
hn=4.5;

%Conductor Horizontal Distances
%---------------------------------------------------------------
din=rad_con;
%---------------------------------------------------------------

%%%% DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
%%%%
%c_length=1000;
zi=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line)));
zin=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(din^2+(hi-hn)^2)));
zni=zni;
zn=(ri_n+RE+j*4*pi*(10^-4)*f*(log(De/GMR_n)));

% REarth = (pi^2*f*10^-4); % [ohm/km] unit length resistance of the equivalent earth return conductor.
% De = 659 *sqrt( rho/f ); % [m] distanza/diametro equivalente ritorno correnti nel terreno
% w=2*pi*f;
%
% zii = ri_line + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_line);
% znn = ri_n + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_n);
% zin = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt(din^2+(hi-hn)^2)));
% zni=zin;
%
%-------------------------------------------------------------------
%%%                          Primitive Matrix
%%
Zabcn=[ zii zin;
     zni znn;]*(c_length)/1000;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
if line_config == 6 % 4 == Service (T-Phase); 25/16 AYCY
    % [Modelled] 1-phase Cable (as per Ciric paper)
    f=50;
    rho=100;
    ri_line=1.18;
    rad_con=0.00279;
    GMR_line=0.7788*rad_con; % ==rad_con*(exp(-0.25))

    % Neutral conductor (concentric)
    K=28; % No. of strands in concentric neutral
    ri_n=1.12;
    rad_nstrand=0.85;
    GMRnstrand = rad_nstrand*(exp(-0.25)); % in feet
    GMR_n = ((GMRnstrand*K*(rad_con^(K-1)))^(1/K)); % equivalent GMR
    of the concentric neutral in feet

    %%
    GEOMETRIC DISTANCES
    %%
    hi=4.5; % Heights chosen arbitrarily - house connections...
    hn=4.5;

    %Conductor Horizontal Distances
    %-------------------------------------------------------------------
    ------
    din=rad_con;
    %-------------------------------------------------------------------
    ------
    %%
    DEFINING THE PRIMITIVE (partitioned) Z-MATRIX
    %%
    %c_length=1000;
    zii=(ri_line+RE+j*4*pi*(10^-4)*f*(log(De/GMR_line)));
    zin=RE+j*4*pi*(10^-4)*f*log(De/(sqrt(din^2+(hi-hn)^2)));
    zni=zin;
znn=(ri_n+RE+j*4*pi*(10^-4)*f*(log(De/GMR_n)));

% REarth = (pi^2*f*10^-4);     % [ohm/km] unit length resistance of
the equivalent earth return conductor.
% De = 659 *sqrt( rho/f );  % [m] distanza/diametro equivalente
ritorno correnti nel terreno
% w=2*pi*f;
%
% zii = ri_line + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_line);
% znn = ri_n + REarth +j*w.*0.46*10^-3.*log10(De./ GMR_n);
% zin = REarth +j*w.*0.46*10^-3.*log10(De./ (sqrt(din^2+(hi-hn)
^2)));
% zni=zin;

%-------------------------------------------------------------------
%%%                          Primitive Matrix
%%%-------------------------------------------------------------------
-----%
%%                  Primitive Matrix
%%
Zabcn=[ zii zin;
        zni znn;]*(c_length)/1000;

end
Bibliography


