Numerical investigation of an inhomogeneous conduction heating process for forging parts

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To whoever rises up after a fall
Preface

This work analyzes an innovative conduction heating process for forging parts. The main idea is to heat up a carbon steel cylinder (42CrMo4) in a non-homogeneous way, to obtain “hot” zones, which reach a temperature round 1050 – 1100 °C, with in the middle a “cold” zone with a temperature that varies between 800 and 950 °C. The main advantage is the possibility to simplify the pre-forming process and to save energy. The most of the work was done using a FEM software, with different configuration of geometry in the area of the contacts and of input in current and frequency applied. The dependency of resistivity, magnetic permeability, thermal capacity and thermal conductivity on temperature are used into the electric harmonic and thermal transient analysis. There are studies for the uniformity of temperature in each interesting direction, but with priority on the transversal surface and then on the longitudinal direction, keeping in count that we want the temperature profile flat on each zone but high local gradients on the transition zone between them. In order to guarantee more generality, some assumption are made: uniformity of distribution of current on the contacts area, only one hot zone instead of two like in the induction model. The first considers a contact conductor without the skin effect when the frequency is raised up. That is because the aim of this work is not to give the best solution to reach our goals in terms of temperature profile, but to give the most general basis to build consideration and choose strategies to development; otherwise this study works only on specific cases depending by external impositions. The second assumption was made because of the effectiveness investigation on the system due to the symmetries on the temperature profile in any direction of interest and for the reduction of time computation. There was not enough time to build the contacts and continue with experimental work, but some tables and simulations
shows what should be the values expected in terms of time and power requirement to reach the results. Only thermal efficiency is calculated because electric efficiency does not keep in count the contact losses that is one of the most important. Thermal results are satisfying, but should be added also the thermal losses due to the conduction of water-cooled contacts. Of particular interest is the working strategy, composed by two processes of heating. In fact the non-homogeneous conductive heating has intrinsically low power and cannot heat also the cold zone up to 850°C in reasonable time. So the first heating could be developed by the direct heating (front-end configuration) as developed in this work, or with other processes like induction, reaching an offset of temperature: all the workpiece should be heated up to the cold zone temperature. The second process is the non-homogeneous conduction heating applied to the hot zones. Due to the local concentration of the power for the geometrical configuration, a system of rotating contacts was developed avoiding hot point and allowing a better temperature distribution, that carry as result time of process comparable to the induction heating system.
Prefazione

Questo lavoro analizza un processo innovativo per il riscaldamento di parti da forgiare tramite la conduzione elettrica. L’idea è quella di riscaldare un cilindro di acciaio (42CrMo4) non omogeneamente, per ottenere zone “calde” che raggiungano temperature attorno ai 1050-1100°C e le rimanenti zone “freddo” con temperature tra 800 e 950°C.

Il vantaggio di questo processo è il risparmio di energia semplificando il processo di pre-forming. La maggior parte del lavoro è stato svolto utilizzando software agli elementi finiti (FEM) con diverse configurazioni parametriche: geometriche modificando l’area dei contatti e elettriche variando i parametri di corrente e frequenza applicate. La dipendenza della resistività, permeabilità magnetica, capacità termica e conducibilità termica in funzione della temperatura sono considerate nell’analisi armonica elettrica e in quella transiente termica. E’stato sviluppato lo studio dell’uniformità della temperatura in ogni direzione spaziale di interesse, con la priorità alla trasversale e poi a quella longitudinale, considerando che si vuole rendere piatto il profilo di temperatura in ogni zona ma si vogliono alti gradienti locali nella transizione tra le zone. Per garantire maggior generalità allo studio, sono state fatte alcune assunzioni: prima l’uniformità della distribuzione della corrente nell’area dei contatti, poi è stata studiata una sola zona calda invece che due come nel modello induttivo. La prima non considera in contributo dell’effetto pelle dei contatti, quando sale la frequenza. Questo perché lo scopo del lavoro non è fornire la miglior soluzione per raggiungere l’obiettivo in termini di profilo di temperatura, ma per dare le più vaste basi per costruire considerazioni e scegliere strategie di sviluppo, altrimenti questo studio funzionerebbe solo in specifici casi dipendenti da imposizioni esterne. La seconda assunzione è stata fatta per l’efficacia investigativa del sistema dovuto dalle simmetrie del profilo di temperatura in ogni direzione di interesse e per la
riduzione del tempo computazionale. Il tempo non è stato sufficiente per sviluppare i contatti e il sistema di alimentazione, ma tabelle e simulazioni mostrano quali dovrebbero essere i valori attesi in termini di tempo e potenza per raggiungere i risultati. L’efficienza termica è stata calcolata senza tener conto le perdite dei contatti raffreddati ad acqua, per questo i risultati sono molto soddisfacenti. Le perdite elettriche e di conseguenza l’efficienza, causa assenza dei contatti, non sono state calcolate. Assume particolare interesse la strategia di lavoro, composta da due processi di riscaldamento. Infatti il riscaldamento non omogeneo ha intrinsecamente bassa potenza e non potrebbe provvedere agli 850°C richiesti nella zona fredda in tempi ragionevoli. Per questo il primo processo potrebbe essere sviluppatto con il riscaldamento diretto (configurazione testa-coda) come realizzato in questo lavoro, oppure tramite induzione, con lo scopo del raggiungimento di una temperatura uniforme su tutto il pezzo. Il secondo processo è quello della conduzione non omogenea, applicato sulle zone da far diventare calde. Dovuta alla locale concentrazione della potenza per la configurazione geometrica, è stato previsto un sistema di contatti rotanti che non permettano la formazione di punti caldi e favoriscano una migliore distribuzione della temperatura sulla superficie trasversale. Utilizzando entrambi i processi si può ottenere un risultato in termini di tempo paragonabile al sistema induttivo.
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Sitography and Bibliography
1. Introduction

“Energy technologies do not just appear from nowhere; they are the results of many year of research work.”
Michael Glos – German Federal Minister of Economics and Technology

1.1 Context: forging processes

Forging is the manufacturing process of forming and shaping metals through localized compressive forces, using hammering, pressing into a die or rolling. The process begins with a stock, or a billet which is heated to its plastic deformation temperature, then upset or "kneaded" between dies to the desired shape and size. The temperature of forging process is important in order to classify it in hot, warm and cold classes. Dealing with steel materials, hot forging is between 1050°C and 1200°C, enough to recrystallize it. It is better to heat up at elevated temperatures because the material is softer to work due to increased ductility, but at the same time dimensional tolerances are more difficult to control after cooling, resulting in lower quality. Cold forging (850-950°C) is done under the recrystallization temperature, so it requires less energy than hot forging and fewer material is wasted; however, the steel requires higher loads to be formed, and formability is limited at low temperatures. It’s preferable to achieve a good compromise between formability, ductility, and accuracy of geometry, with a final temperature around the crystallization point (700-950°C for steel).

Exist several processes to forge materials, using diverse ways to shape the

**Figure 1.1** a) Open die forging. b) Cogging forging  c) Closed die forging
material as illustrated in picture below. There is the open and closed die forging, extrusion and other starting from these. From here the reference will be to open die forging mostly: the metal is shaped by hammering or pressing between flat or simple countered dies. While impression or closed die forging confines the metal in dies, open die forging is distinguished by the fact that the metal is never completely confined or restrained in the dies. High-strength, long-life parts optimized in terms of both mechanical properties and structural integrity are today produced in sizes that range from a few pounds to hundreds of tons in weight.

In forging processes pieces are heated and forged into a preform, then heated once again and forged into the end – form. To achieve this process, special machines, like rollers, are needed to forge the blanks into a preform. Especially in small and medium companies it is not possible to integrate rollers cost efficiently into their production processes due to their small amount of production batches. From the technical point of view, the preform helps to save material in the forging process. Although this kind of heating has been so far only subject of research, its industrial applications are remarkable, and involve mainly forging processes of complex geometry such as crankshaft and camshaft.

![Figure 1.2 example of stages of forming process](image)

We want to achieve a good distribution of the mass inside the form, without using special machines. This solution can be reached adopting an “inhomogeneous heating” of the batch. The parts of the forging billet, which are supposed to have
a high material flux inside the form, are heated until 1050 - 1200 °C. The parts of the forging work-piece that are supposed not to have such a high material flux inside the form reach instead “colder” temperature between 800 and 900 °C. Once the piece has been heated up, compressive forces are applied with consequent different behaviors that depend on temperature. An example of an “inhomogeneous” heating is displayed in Fig. 1.3. A further advantage of this approach is the decrease of the amount of energy used, due to the fact that some parts are not heated as strong as in the case of homogeneous heating.

![Figure 1.3 Open die forging in the case of “inhomogeneous” heating](image)

1.2 Approach to the problem

The problem refers to the electrical and thermal analysis, and a study on induction technology has been already carried out. The results of these models and simulations will be compared in the last chapter of this thesis. The billet is a cylinder of 42CrMo4 (carbon steel) 474 mm long, with a diameter of 30 mm. The main goal is to investigate over the parameter configurations for non-homogeneous conduction heating, to obtain the squared temperature profile required, with hot and cold zones divided by local jumps. In Fig. 1.4 is shows a view of the billet, the red part is the hot zone and the orange the cold one.

![Figure 1.4 Explanatory sketch of the distribution of temperature along the main direction](image)
It is a fact that a current flowing in a conductor warms it due to the joule losses, which are related to the square of the current, and also to the resistance of the conductor (1.1). The joule losses generate heat where they are present, so they are the most interesting electrical parameter to study, that connect thermal and electrical problem. From this point we can stop over two argumentations:

- In (1.2) First the resistance R is a parameter proportional to the resistivity, so an electrical property of the material, and with length and inverse to the section, so geometrical properties. This relation is known as “second Ohm’s law”, but it works only with cylindrical conductors. Our billet has the right shape, but the terminals are not placed at front-end configuration as required; otherwise, we would obtain a direct heating, and that would mean uniform temperature without hot and cold zones as required. So, we are ready to understand that the geometrical section and length of the paths followed by current could concentrate the joule losses where the sections are smaller, privileging shorter paths to make less effort.

\[
P \ [W] = R \cdot i^2 \quad \text{(1.1)}
\]

\[
R \ [\Omega] = \rho \cdot \frac{l \ [m]}{s \ [m^2]} \quad \text{(1.2)}
\]

- Secondly, the current has a different behavior in modifying the frequency, due to the skin effect: the tendency of an alternating current density to concentrate near the surface of the conductor, and decrease exponentially with greater depths in the conductor. The skin effect causes the effective resistance of the conductor to increase at higher frequencies where the skin depth is smaller, thus reducing the effective cross section of the conductor. The skin effect is due to opposing eddy currents induced by the changing magnetic field resulting from the alternating current. In the first skin depth layer, flows the 63.2% of the total current, in the first three layers above the 99%. This fact illustrates why this behavior is important in our system.

\[
\delta = \sqrt{\frac{\rho}{\pi \cdot f \cdot \mu \cdot \mu_0}} \quad \text{(1.3)}
\]
This is valid until the steel is magnetic, that means under the Curie temperature (760°C). Passed this point $\mu = 1$, the current is distributed more homogeneously in the section of a cylindrical conductor, distributing on the section predominantly in function of resistivity.

1.3 Input and output

The input parameters are both geometrical and electrical:

On one side, we should design the best configuration for the contacts that will heat the billet, on the other side we have the electrical parameters, that means more degree of freedom to be chosen. In a view of optimization or simpler as a parameter study, to reach a set of suitable values, we need to consider four design variables as input:

- The geometrical parameters for the contact design are two, because we consider the case in which the area of the contact is rectangular shaped, attached to the surface of the cylinder. For this reason, the area is not planar, but we can consider the two parameters as an input, as shown in Fig 1.5, representing a symmetric section of hot zone:
  - y_angle: this value is referred to the angle related to length of the arc along the azimuthal direction, because it is easier deal with a cylindrical coordinate, as explained in chapter 3.
  - z_curr: this will be the depth along z direction of the contact, where the current flows.

- The electric parameters are also two and, as a following of the imposition, all the other result consequently. We were careful to stay in the field of usable values at ETP, but also in an industrial application:
  - Current: prefixed value imposed on a surface (determined by the geometrical parameter y_angle and z_curr) is for generality but also a limitation of this model.
  - Frequency: prefixed value imposed as the same with the current, allows the harmonic analysis, with all the simplification of the case compared to others electrical analysis.
All the range values used for the parameters will be discussed below, but it is important to notice that the area value used for the simulation will be 4 time higher in the reality, for symmetry.

The current is applied on the surface directly over the billet because the electrical contacts are not considered in the study. Of course, they are necessary to develop a complete study and the current will be concentrated on the edge due to the frequency, so the current will be no longer equally distributed on the surface. Nevertheless, this work does not want to give to the user the best final solution, but rather more solid halfway solutions, which can be the groundwork from where other considerations can start. In a real application, goal temperature is also affected by other aspects, like the transport time required from the place where the heated and where it is formed.

The output parameters are all temperature measures; then we will measure the results at the end of the thermal simulation in function of the reaching of temperatures of cold and hot. The four temperatures measured have been chosen as a compromise between the amount of data that could be analyzed point by point in the simulation, and the real temperature that could be measured in a laboratory experiment. The position of measured temperature points are illustrated in figure 1.5.
There are four points of measure \((T_1, T_2, T_3, T_4)\) that are related to 3 indicators, which are ranges of temperature \(\Delta T_1, \Delta T_2, \Delta T_3\) in each cartesian direction (x,y,z):

<table>
<thead>
<tr>
<th>(\Delta T_1)</th>
<th>Surface hot top – core hot</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta T_2)</td>
<td>Surface hot top – surface top cold</td>
</tr>
<tr>
<td>(\Delta T_3)</td>
<td>Surface hot edge – core hot</td>
</tr>
</tbody>
</table>

The purpose of these measures is to investigate over the uniformity of the temperature. In this study, we will see how is not so interesting the indicator \(\Delta T_3\): it is intrinsically little, instead we will focus more on the \(\Delta T_1\) to reach our results.

It was not possible to follow also the improving of the temperature \(\Delta T_2\), challenge left for the future. The previous induction model was characterized by 2 hot zones of 70 mm each; the inductor was shaped to have more coils above the hot zone, where the heating sources concentrate non-homogenously along the main direction.

1.4 Constraints and goals

The constraints are really important to set limits to fields of investigations in advance: they allow to impose the right initial conditions not only for the billet, but also for the environment, and to set the material properties in a range of
specified values of simulations. It is important to understand which are the most influencing parameters changing to take into account, to simulate properly the reality: i.e. we take into consideration the variability of the resistivity in function of temperature, but we consider constant the volume of the billet without analyzing the thermal elongation.

Most important constraints and requirements:
- 1400°C temperature of steel melting: it cannot overcome locally, otherwise the properties of steel change so far from the expected and could happen a separation of mass.
- 760°C Curie temperature is not strictly a constraint but a key point to take into account, because over it the work-piece will be no more magnetic, so the transferred power will be lower. This phenomenon is more evident in the case of induction heating, but also conduction need to be managed accurately.
- Hot zone must have a temperature in all its volume in the range of 1050-1200°C, 70mm wide in the main direction.
- Cold zone must have a temperature in all its volume in the range of 850-950°C, all the remaining volume. A transition zone, as short as possible, is accepted, and there are no specific constraints.
- Use of 4500A and 3000Hz as current and frequency parameter, for the best use at ETP. To overcome these values means to stress the laboratory equipment, but most important, it means to simulate values that cannot be used commonly by the companies. So, it is necessary to set also this limit for further considerations.

Goals require strong conditions, not simply the reach of temperatures in the zone. To establish if a simulation - reaching the requirements of heating - is better than another, the judgment is done by:
- Uniformity of temperature in the zone, using the objective parameter of previously defined indicators $\Delta T_1, \Delta T_2, \Delta T_3$.
- Minimization of time process, keeping in mind that induction heating lasts about 80-100 seconds. For this reason, we look for a conduction heating
process of approximately that time or little more, in the range of some minutes, considering that induction can transfer more power when steel is still magnetic.

1.5 Following work

In next chapters, the numerical analysis is conducted using the commercial FEM simulation software ANSYS©. The simple geometry of the work-piece suggests using a 2D axisymmetric model to reduce the computational cost, but due to the disposition of the conduction current on the transversal surface (radial plane of the billet) we must use a 3D model to simulate properly the behavior of the system.

Special attention will be paid on the code, specifying some interesting points and some user knowledge, useful to a smooth entry into the line programming mode. Indeed, learn how to code and all the right commands required a strong effort, even having already used other FEMs like Flux and Comsol.

Once built up a model that meets our needs, we will investigate the effectiveness of the heating and how it could be improved, analyzing the results. If they satisfy the requirements, a comparison is done to see how these solutions fit into the goal parameters doing the first parameter study.

After the parametrical study, it is clearer how to improve the model, choosing appropriated parameters; modifying them is understandable how they are affect the results. In this way, the field of investigation is restricted, and it is possible to achieve better simulations, until a set of “good solutions” is reached and further presented. At last we will choose a solution from this set, that is comparable in terms of time and power to induction heating.

![Figure 1.7 Block diagram of logical investigation outside the software](image-url)
Now that the investigation steps are explained, the subdivision inside the software is more similar to next scheme:

![Diagram](image)

**Figure 3.1** – Scheme to solve a problem with a FEM software

This subdivision is more appropriate to follow the program steps, so this will be the configuration followed in this work in next chapters, as follow from chapter 2:

2. Theory of conduction heating: This is the idealization part. Starting from the physical problem, we need to understand the dominating laws and the condition under that develop a mathematical model, that is not the real one but one that behaves like the real one, with also some possible simplification. This is a work usually made with books, pen, paper and calculator.

3. The model and the code: Then comes the discretization step, where we can find our coding focus. Here, all the mathematical model is transformed into code (or commands) to build the discretized geometry, then the mesh, material properties and loads conditions. There is all the necessary for setting the right problem in a good way (i.e. using some symmetries is smart to get same result with half of time calculation).

4. Solution and first results: The solution is obtained from the discrete model and then analyzed with the postprocessing part of the software. Here the problem is solved with numerical method, i.e. a system of ordinary and partial differential equations (PDE). Before this step we do not know if the results are good or not. Here we have to analyze data and then choose how to modify the previous model
to get better results, or to change some imposed condition, or change some parameter to investigate the behavior of the system in different conditions.

5. Second results: Here there will be a change of approach in the working strategy. Only the non-homogeneous conduction heating cannot be an effective process if used alone, but is good at shaping temperature because has really local affection. Is developed the front-end connection to heat the work-piece till the cold zone temperature requirement and then the non-homogeneous heating shapes the hot zones. After considerations from the practical tables built for direct heating, is presented a second parameter study.

6. Comparison conduction and induction: This last part take into account the results in different terms of the two simulations, comparing number like energy consumption, time, power, and also advantages and disadvantages between them, like versatility, electric system requirements, maintenance, that should be considered when a company decides to use one of these two technologies.
2. Theory of conduction heating

“There is Nothing More Practical Than a Good Theory”
Kurt Lewin – German-American psychologist, father of social psychology

The theory here presented is useful to describe the phenomena that occur in conduction heating. It will be also applied, as developed here, in the second part of chapter 5. However, direct resistance heating applied to our work-piece will be different from the usual one, due to the high variation of geometrical transversal sections during the flow of the current in a non-homogeneous way.

2.1 Electromagnetic and thermal problem

Every electromagnetic problem can be fully described by Maxwell’s equations. This set of equation is called also “full” because it can solve any electromagnetic problem, and it is the most general. However, we are looking for the easier case of investigation avoiding useless complications, so some equations will be simplified. Nevertheless, we cannot simplify too much, otherwise some useful information will be forgotten.

Here the differential Maxwell’s laws are presented in many form, starting from the “full”. The others are simplifications made to study simpler problems:

<table>
<thead>
<tr>
<th>Full Maxwell</th>
<th>Static case</th>
<th>Magneto quasistatic</th>
<th>Electro quasistatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nabla \times E = -\frac{\partial B}{\partial t}$</td>
<td>$\nabla \times E = 0$</td>
<td>$\nabla \times E = -\frac{\partial B}{\partial t}$</td>
<td>$\nabla \times E = 0$</td>
</tr>
<tr>
<td>$\nabla \times H = J + \frac{\partial D}{\partial t}$</td>
<td>$\nabla \times H = J$</td>
<td>$\nabla \times H = J$</td>
<td>$\nabla \times H = J + \frac{\partial D}{\partial t}$</td>
</tr>
<tr>
<td>$\nabla \cdot B = 0$</td>
<td>$\nabla \cdot B = 0$</td>
<td>$\nabla \cdot B = 0$</td>
<td>$\nabla \cdot B = 0$</td>
</tr>
<tr>
<td>$\nabla \cdot D = \delta$</td>
<td>$\nabla \cdot D = \delta$</td>
<td>$\nabla \cdot D = \delta$</td>
<td>$\nabla \cdot D = \delta$</td>
</tr>
</tbody>
</table>

Table 2.1 – differential Maxwell’s equations for different problems
The differential formulation is valid punctually, instead the integral one possible reachable by manipulating the same laws, is valid as integral form, so with less mathematical restriction. Anyway, engineers deal with integral quantities (current, voltage) so in the end everything must be converted into integral quantity. For this reason, the two approaches are the same, but we decided to show the differential path.

The first \( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \), also known as Faraday-Neumann-Lenz is the law that introduces the non-conservative electric field, produced by a time-varying induction field. It regulates the behavior of all electrical machines: static as transformator for the variation of the magnetic flux in the time and also rotating machines as an electric motor, that converts electrical energy into mechanical and vice versa for the variation of the perpendicular area in the time. Vector induction \( \mathbf{B} \) is related to the magnetic field \( \mathbf{H} \) by the constitutional law, but this one is the product of magnetic flux and the perpendicular area where the flux is flowing. Changing one of this parameter in the time cause the creation of an electromotive force [V].

The second law, \( \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \) is the Ampere’s law, even if the partial derivation term was added by Maxwell, otherwise the law does not work in displacement current inside a capacitor. This law is related to the creation of the magnetic field from a non-magnetic source. If a current flows in a wire, a magnetic field is created around it; if this wire is also coil-shaped, we will obtain an inductor, the basic component related to reactive inductive power.

The third law, \( \nabla \cdot \mathbf{B} = 0 \) reminds in all cases that the magnetic field cannot create energy, but only convert it, store and transport it. Every circulation of the magnetic field is 0, made by closed lines, and it cannot do any type of work.

The Gauss’s law is \( \nabla \cdot \mathbf{D} = \delta \), and it allows to calculate the free charge in a volume.

By combination of these four laws, also the charge conservation law can be obtained, called also continuity law: \( \nabla \cdot \mathbf{J} = -\frac{\partial \delta}{\partial t} \). It specifies that the charge is not only conserved in space, but dynamically in time, when a current is flowing.
The other cases are simplifications: the static case is used to find value of electric and magnetic field of constant value, indeed static. Magneto and electro quasistatic neglect the displacement current, due to the low “electrical length” of the system analyzed. This indication is used to understand if the system propagate waves like an antenna or receive radiation coming from outside. The electrical length $\Lambda$ is the dimension to be compared to physical biggest dimension ($L$) in this way:

$$\Lambda = \frac{L}{\lambda} = \frac{f \cdot L}{c_0}$$

with the characteristic wavelength $\lambda$, frequency ($f$) applied to the system, $c_0 = 3 \cdot 10^8 [m/s]$ the speed of the light. If $\Lambda < \frac{1}{10}$ or in general little, it means that the electrical system considered is small: it does not irradiate and is not affected by radiation of $\lambda$ wavelength. Only in this case the component of radiation the displacement current $\frac{\partial D}{\partial t}$ can be neglected.

Our problem is a magneto quasistatic case, so indeed we can neglect the displacement current, but we need to keep in consideration the variation of magnetic field, as a simple conclusion of our knowledge.

We need also the constitutive laws, that relate the induction with magnetic field, displacement and conductive current with electric field, underline the property of materials.

$$B = \mu \cdot H = \mu_0 \cdot \mu_r \cdot H$$

$$D = \varepsilon \cdot E = \varepsilon_0 \cdot \varepsilon_r \cdot E$$

$$J = \sigma \cdot E = 1/\rho \cdot E$$

$\mu$ is the magnetic permeability, that measures the ability of a material to support the formation of a magnetic field within itself. It is made by the two components: $\mu = \mu_0 \cdot \mu_r$ with $\mu_0$ the magnetic permeability of the vacuum: $4\pi 10^{-7} [H/m]$ and its relative $\mu_r$. 
\( \varepsilon \) is the electric permittivity, or the measure of resistance that is encountered when forming an electric field in a particular medium. It is made by the two components: \( \varepsilon = \varepsilon_0 \cdot \varepsilon_r \) with \( \varepsilon_0 \) the vacuum permittivity: \( 8.85 \cdot 10^{-12} \, [F/m] \) and its relative \( \varepsilon_r \).

\( \sigma \) is the electric conductibility, that is the reciprocal of electrical resistivity \( \rho \), and it measures a material's ability to conduct an electric current \( [1/(\Omega m)] \).

Often it is not sufficient to describe the material property as a constant. Indeed, this is only the easiest condition, when the material is “OIL”:

- Homogeneous: the material property does not change in spatial position
- Isotropic: the mutual directions of the tensor of the property (values outside the diagonal) are zero – each direction is independent from the others.
- Linear: a variation as input of the imposing magnitude, the material answers with a proportional change (i.e. deformation of a spring in elastic zone when a force is applied).

There are also hysteresis phenomena that depend on the history of the material, but in this work they will not be considered. In real applications, no one of these conditions are satisfied, but is possible to work in some zones where they are true. Sometimes it is useful to consider the variation of a quantity with no influence to affect results in that field of evaluation. Also know which dependencies are the most influencing change the tasks of the problem: we are in magneto quasi-static case, where electric resistivity \( \rho \) and magnetic permeability \( \mu \) will be used. In this case, we take in consideration their dependence from temperature \( \rho(T), \mu(T) \). Non-linearity graphs of material property dependence will be shown further in this chapter.
2.2 Direct resistance heating

Called also front-end heating because of the geometrical configuration, direct resistance heating is based on the flow of an electric current through an electrically-conductive body, which is directly connected to an electrical supply. The work-piece is heated by Joule losses, due to the internal resistance referred to the law (1.1). This technology enables fast heating of metals, thereby contributing to improve the heating efficiency and realization of a clean work environment.

The heating can be realized by DC or AC current, but the industrial applications are mostly done with the alternating one. When a DC current flows in a straight shaped body of constant electrical resistivity and constant cross-section, the current density and, as a consequence, the power density due to Joule’s effect is uniform in the cross-section. Non-uniform distribution of the current density can arise only if the body has not a straight shape, or the dimensions of its cross-section vary along the body length as in our application.

When an AC current flows in the work-piece, the current density and internal power sources distribution will be determined by the same laws mentioned in the previous paragraph, i.e. the law of electromagnetic induction, the Ohm’s law and the Joule-Lenz law. Other phenomena also have considerable influence on the power density distribution like:

- skin-effect, already explained in law (1.3)
- proximity effect, where a current flows when there are several conductors, one near to each other
- slot effect, due to magnetic flux concentrators, that modifies the current density distribution inside the conductor
- ring effect in not straight conductor, when the current accumulate itself in the shorter path

When temperature increases in the process of heating, usually electrical resistivity increases like in our case. That creates a non-uniform distributions of the current density and internal heating sources, both in the cases of AC or DC current supply. For this reason, we will conclude, in our case of study, that AC direct heating is preferable.
2.2.1 Advantages and limits

There are some evaluable advantages about the direct heating compared to other processes that need to be taken into account:

- high energy efficiency
- fast heating processes
- high specific heating power
- high production rate
- automation of the process
- clean working environment

There are also some limit issues that affect the direct heating versus other processes:

- only electric conductor material can be heated up
- the work-piece needs to have a simple geometry, with constant section and high ratio between length and radius
- requires high current

The first aspect is not so interesting for our investigation, because it is a limit overcome by other technologies with external heating sources, like resistors furnaces or fuel burners; anyway in this study will be considered mostly internal sources technologies.

The geometry limit is the one of the two biggest limits for resistance conduction heating versus induction, because using induction can be studied where to concentrate the magnetic field with an external coil, instead in conduction heating the power is concentrated where the resistance is bigger, due to the physical connection of terminals that feed current to the body. For the geometrical remind we have to look the second Ohm’s law (1.2). The third limit is an important problem due to the nature of the heating: a metallic material has a low resistivity, and its resistance is low if the length is short and the section is big. To heat it up, it needs power like explained in the law (1.1) but if the resistance is low, the current should be big, even if has the magnitude of two.
They prefer to avoid the problem, accepting a bigger power dimensioning solution like induction, due to the higher reactive power required. For example, is considered the heating of a bar with a variation of the diameter in the middle like in Fig. 2.1

![Figure 2.1 - Heating of a bar with a non-uniform cross section](image)

Using 1 and 2 as pedix to identify the two regions, we can deduce some useful information as following:

\[
\begin{align*}
\frac{R_2}{R_1} &= (1 + \varepsilon) \\
S_2 &= S_1 (1 + \varepsilon)^2 \\
J_2 &= J_1 / (1 + \varepsilon)^2 \\
w_2 &= w_1 (1 + \varepsilon)^4 \\
\Delta\theta_{m2} &= \Delta\theta_{m1} (1 + \varepsilon)^4
\end{align*}
\]

Where \( R \) is the radius \([m]\), \( S \) the cross section \([m^2]\), \( J \) the current density \([A/m^2]\), \( w \) the specific power losses \([W/m^3]\) and \( \Delta\theta \) the jump of temperature \([^\circ C]\). At last, for little values of \( \varepsilon \) we can do a first order Taylor series approximation:

\[
\Delta\theta_{m2} = \Delta\theta_{m1} (1 - 4\varepsilon)
\]

From this step results the unevenness of the heating, also with small variation in diameter, that limits the use of this process only to bodies with regular cross sections, i.e. if \( \varepsilon \) is 3% the difference of temperature is above 10%. If we use as following the non-homogeneous conduction heating, we will expect that the heat will concentrate where the section is smaller, in the region near to contacts.
2.2.2 Thermal transient

The study of thermal transient is necessary to know the thermal behavior, with a given power density to the billet, how the temperature grows and how the environment conditions affect the raising. We will not go deep into details, but a general approach will be followed. So here is reported the differential equation of thermal conduction where there is in one side internal generation of heat and in the other side the losses:

\[ \nabla^2 \theta(P^*, t) - \left( \frac{c \gamma}{\lambda} \right) \frac{\partial \theta(P^*, t)}{\partial t} = - \frac{w(P^*, t)}{\lambda} \quad (2.1) \]

Where \((P^*, t)\) means that the referred temperature \(\theta\) or power \(w\) are variable of space \(P^*\) and time \(t\). \(\lambda\) is the thermal conducibility of the material \([\text{m}^2 \text{C} / \text{K}]\). If \(\frac{\partial \theta}{\partial t} = 0\) then the solution of the equation will be a stationary field of temperature. For the general solution of the equation, with the hypothesis of \(w=w_0=\text{const}\) and initial conditions of uniform temperature the general equation is converted in cylindrical coordinates and some adimensional variables are introduced to reach a solution independent form time and radius. In Fig. 2.2 the behavior is shown as example of a heating of steel bar from 0 to 1250°C, keeping into account the thermal losses to the environment.

Since the heat sources are homogeneously distributed in the section of the body and the thermal losses are considered, the temperature at the axis is always higher than the surface one. We will not go into detail with this study, because the previous considerations are developed from a simplified model that does not keep into account the changing of materi- al properties. For this reason, the influence of variation of characteristics of the material with the temperature, affects considerably the results in all heating processes. This method is only available for limited temperature ranges, where a constant value for properties can be considered.
2.3 Heating of cylindrical conductors

In this paragraph, we will study the behavior of heating with alternating current, focusing on the distribution of them: it is an aspect that will affect all the following simulation work.

2.3.1 Skin-effect in cylindrical conductors

The skin-effect is one of the principal effects in this investigation, therefore it must be explained. It produces a non-uniform distribution of the AC in the cross section of a conductor. The current density decreases from the surface towards the internal part of the body. In a conductor in which DC current flows, the current density \( J \) is constant in the cross-section and is given by the ratio:

\[
J = \frac{I}{A} \left[ \frac{A}{m^2} \right]
\]

with \( I \) the current in the conductor, \( A \) the cross section of the conductor. A different situation occurs in the conductor if an AC current flows, as illustrated in Fig. 2.3a, where a current \( I \) is applied to in an infinitely long metallic cylinder. The rule of the right-hand screw determines the direction of lines of magnetic flux.
density $\mathbf{B}_t$ produced by the current $I$. The AC magnetic flux density $\mathbf{B}_t^*$ induces in the conductor the eddy current $i$, which in turn produces a reaction magnetic field, characterized by the vector $\mathbf{B}_t$.

According to Lenz’s law, the magnetic field $B_t$ opposes itself against the process that creates it. So, the direction of the magnetic induction vector, produced by the current $i$, will be opposite to that of the magnetic induction created by current $I$. The direction of the path of the induced current $I$ is calculated with the right-hand screw law. Therefore, the induced eddy currents in the conductor $i$ will increase the total value of the current near the surface, and decrease it in the central part of the conductor; this behavior produces an unhomogeneous distribution of current density in the cross section. This phenomenon is qualitatively illustrated in Fig. 2.3b.

In the case of a pronounced skin-effect (or in the conducting semi-infinite body), the reduction of the current density from the surface towards the interior of the conductor follows the exponential law:

$$J = J_e e^{-\frac{y}{\delta}}$$  \hspace{1cm} (2.3)

where $J$ is the current density at a distance $y$ from the conductor’s surface $[A/m^2]$; $J_e$ is the current density at the conductor’s surface $[A/m^2]$; $\delta$ is the already known penetration depth of the electromagnetic wave $[m]$; $\delta$ is a quantity.
with dimension of a length, which characterizes the degree of attenuation of electromagnetic field in the conductor.

According to Eq. (1.3), the value of the penetration depth varies according to the square root of electrical resistivity and inversely with the square root of relative magnetic permeability and frequency. Therefore, it depends on both the electromagnetic properties of the conductor’s material and the applied frequency, and at the same frequency, it has different values in different materials. Since the volumetric power density depends on the square of J, according with Eq. (2.1), its distribution from the surface towards the interior of the conductor can be expressed by the equation:

\[ w = w_e e^{-2y/\delta} \]  \hspace{1cm} (2.4)

with \( w_e = \rho J_e^2 \). The curves of Fig. 2.4 show the commonly assumed classical distributions of current and power density versus the distance from surface (in current penetration depths) due to the skin effect, obtained from (2.1) and (2.2). The analysis of the curves shows that at a depth greater than 3 times \( \delta \), the values of all the quantities of interest (current density, volume specific power and also magnetic field intensity) are practically negligible.

**Figure 2.4** Current density \( (J) \) and power density \( (w) \) distributions as a function of the distance from the surface of the work-piece.

Now it is easier to understand the arguments developed in the first chapter:
substituting \( y \) with \( \delta \), the (2.2) becomes \( w = w_e e^{-2} \), so the power concentrated in the first layer is \( \frac{w_e}{w} = \frac{1}{e^2} = 0.865 \). In this way, we have demonstrated that 86.5% of the power in AC heating is concentrated in the first layer of \( \delta \).

The considerations for the current is almost the same, with the difference of the square, obtaining the concentration in the first layer of 63.2% of the current.

The skin effect is a good argumentation to introduce the problem, but now, starting from Maxwell equations, we will examine it in depth.

### 2.3.2 Distribution of current and power

In the direct resistance heating of “long” cylindrical work-pieces, electrical and magnetic quantities (current density, electric and magnetic field intensities, internal power sources) have one-dimensional radial distribution in the cross-section. We consider now the flow of an alternating current in a metallic homogeneous cylindrical work-piece with the following assumptions:

1. The work-piece has infinite length. In this way the end effects, due to the uneven current density distribution near the contact system, are neglected. All field quantities, current density, electric and magnetic field intensity are assumed to be dependent only on the radial position.

2. The same quantities are sinusoidal in time.

3. Relative permeability \( \mu_r \) and resistivity \( \rho \) of the work-piece are constant.

In these hypotheses, Maxwell’s equations (Table 2.1) can be written as:

\[
\text{rot} \ H = \frac{E}{\rho} \quad ; \quad \text{rot} \ E = -j\omega\mu_0\mu_r H
\]  

Where \( H, E \) are the complex magnetic field and electric field intensities measured in \([A/m]\) and \([V/m]\), \( \mu_r \) is the relative magnetic permeability; \( \mu_r \mu_0 = 4\pi 10^{-7} \) is the magnetic constant of vacuum \([H/m]\); \( \omega = 2\pi f \) is the frequency pulsation of current, \([\text{rad/s}]\) and \( f \) the frequency \([\text{Hz}]\).

In cylindrical coordinates system, Maxwell’s equation can be rewritten as follows:
And, from the second equation, we can write:

\[
\frac{dH}{dr} + \frac{1}{r} H = \frac{E}{\rho} \tag{2.6}
\]

\[
\frac{dE}{dr} = j\omega\mu_0\mu_r H \tag{2.7}
\]

So, considering the first equation, knowing that \( E = \rho J \)

\[
\frac{d^2 J}{dr^2} = \frac{1}{r} \frac{dJ}{dr} - j \frac{\omega\mu_0\mu_r}{\rho} J = 0 \tag{2.9}
\]

Introducing the following dimensionless parameters, we have:

\[
\xi = \frac{r}{r_e}; \quad \beta^2 = -j \frac{\omega\mu_0\mu_r}{\rho} r_e^2 = -jm^2; \quad m = \frac{\sqrt{2}r_e}{\delta}
\]

We have just introduced \( m \), one of the most important parameter in electrical heating processes, because is related with geometrical parameter (radius \( r_e \)) and with electromagnetic one (\( \delta \)). Knowing only this parameter, it can be possible to make some considerations over the general heating process, without any constraints.

The Eq. (2.6) can be written in the dimensionless form:

\[
\frac{d^2 J}{d\xi^2} + \frac{1}{\xi} \frac{dJ}{d\xi} + \beta^2 J = 0 \tag{2.10}
\]

The solutions of Eq. (2.10) are:
\[ J = \hat{C}_1 J_0(k\xi) + \hat{C}_2 Y_0(k\xi) \]

With \( J_0, Y_0 \) Bessel functions of order zero, of first and second kind. The integration constants \( \hat{C}_1 \) and \( \hat{C}_2 \) can be determined with the boundary conditions:

\[
\begin{align*}
J &\neq \infty \text{ for } \xi = 0 \\
J &= \infty \text{ for } \xi = 1
\end{align*}
\]

Since it is \( Y_0(0) = \infty \), it must be \( \hat{C}_2 = 0 \), while for \( \xi = 1 \) it is \( \hat{C}_1 = \hat{j}_e/\hat{j}_0(k) \), with \( \hat{j}_e \) that is the complex value of the current density at the surface of the cylinder. Substituting we obtain:

\[
J = \hat{j}_e \frac{J_0(k\xi)}{J_0(k)} = \hat{j}_e \frac{J_0(\sqrt{-jm}\xi)}{J_0(\sqrt{-jm})} \tag{2.12}
\]

The equations above show that when the electromagnetic wave penetrates inside the cylinder, both module and phase angle of the current density vary.

**Figure 2.5** Change of module and phase angle as a function of the distance \( x \) from the surface, for different values of \( m \).
Figure 2.6 a and b give the diagrams of the relative current density and relative power density along the radius for different values of m. For $m \leq 1$ the distribution of the current density is practically uniform, like in the DC case.

For further investigation on electrical side, to calculate the equivalent impedance - the internal reactance per unit length - it is advisable to refer to the coefficients $k_r$ and $k_x$. These two separate the active component of the power (DC) to the reactive one, keeping into account the influence of skin-effect. There are formulas and tables not reported here because the part of electrical efficiency was not developed here, due to the lack of specification of the contacts.

### 2.3.3 Heating of magnetic steel rods

As stated before, the exponential distribution of current previously described occurs in a homogeneous and linear body in case of pronounced skin effect. At lower frequencies, and also in case of linear and homogeneous materials, the distribution differs from the exponential one depending on the
frequency, the material characteristics and the geometry of the body to be heated.

In many applications of induction heating or direct resistance heating, during the thermal transient, in different points of the cross-section, the temperature may have different values. This is due to the skin effect and to the local resistivity, that depends from the temperature, varies from point to point. This affects the distribution of the specific power per unit of volume, given by Eq. (2.4), and the distribution of heat sources in the cross section, which may differ significantly from the exponential distribution even in case of pronounced skin effect.

This phenomenon is more pronounced in ferromagnetic work-pieces, where the current density distribution is influenced not only by the temperature dependent resistivity, but also by the local values of magnetic permeability, which depend not only on the temperature but also on the local magnetic field intensity. Figure 2.7 shows, for example, how the radial distribution of specific power modifies during direct resistance heating of a magnetic steel rod.

![Figure 2.7 Radial distribution of power sources at different temperature in a ferromagnetic steel rod](image)

In the graph, the radial distribution of specific power per unit of volume is plotted against the radial distance from the center of the rod. The temperature dependence of the specific power is shown for different values of the temperature 

\[ \theta_S = 25^\circ C \]

and

\[ \theta_S = 1250^\circ C \]

The specific power is given by

\[ W/m^3 = \frac{2r}{d} \]

where \( r \) is the radial distance and \( d \) is the diameter of the rod.
The curves, which relate to different rod’s surface temperatures, show that at the beginning of the heating the power distribution is nearly exponential. In the following instants of heating transient, the power distribution remains highly non-uniform below Curie point, due to variations of resistivity and permeability with temperature and local magnetic field. There is a drastic change above the Curie temperature, where the distribution becomes more “flat”.

A singular phenomenon, known as “magnetic wave”, occurs in induction surface hardening where the power density distribution along the radius/thickness may show a peculiar shape, significantly different from the commonly assumed exponential distribution. The power density here has its maximum value at the surface, and decreases toward the core. But then, at a certain distance from the surface, the power density suddenly starts to increase again, reaching a maximum value before it finally declines.

![Figure 2.8](image)

**Figure 2.8** a Schematic representation of the magnetic wave phenomenon; b radial distribution of power density in induction heating at 4 kHz of a carbon steel shaft, 100 mm diameter, with $H_0 = 240 \text{ kA/m}$ (rms value)

This consideration is mostly done for induction processes, but here with conduction has the same behavior due to the exponential distribution of heating sources. In surface hardening, carbon steel retains its magnetic properties in the core (region 2 in Fig. 2.8a), while a surface layer (region 1) which is at higher temperature, becomes non-magnetic being heated above $Ac_3$, the critical
austenitization temperature for steel.

In this situation, the current density distribution can be roughly schematized with two exponentials corresponding to two different values of $\delta$ caused by the material characteristics of the magnetic and non-magnetic regions, as it is illustrated in Fig. 2.8a. In particular, in some cases, the value of current and power density, in correspondence to the surface of separation of the two layers, is higher than the one on the surface.

However, given that the values of $\rho$ and $\mu$ gradually change from layer to layer, the result of distributions are more complex, as indicated qualitatively by the dashed curve in the same figure. The phenomenon can manifest itself with the maximum values in an internal layer of the work-piece and not at its surface. Figure 2.8b shows an example of the radial distribution of power density.

In the first part of the graph, with low resistivity and high magnetic permeability, the heating is done with values $m > 1$, so with unevenness distribution of power sources and higher temperature on the surface.

The second part is characterized by high resistivity and magnetic permeability $=1$, so the heating is done with values $m < 1$. The distribution of power sources is more homogeneous, and this time we must consider also the thermal losses, previous neglected.

![Figure 2.9](image_url)  
**Figure 2.9** a Thermal transient of heating of steel rods  
$\theta_s$ = surface temperature, $\theta_a$ = axis temperature
In Fig. 2.9, we can see an example of the behavior of the temperature on the axis and on the surface. We must be careful with the "volcano effect": if we overcome the melting point temperature, it could be possible that the surface is solid, but there is melt metal behind it, that can deplete the properties of the material also for further thermal processes.

### 2.3.4 Thermal efficiency

One of the most important parameters in direct current heating applications is the thermal efficiency of the process. It is defined by the ratio between the thermal energy used to increase the temperature of the material $P'(t) = c\gamma \pi R^2 l \Delta \theta_m$ at a given temperature, and the total heat energy given to the piece, converted into the Joule $P(t)$:

$$\eta_t = \frac{\int_0^{t_0} P'(t) \, dt}{\int_0^{t_0} P(t) \, dt}$$  \hspace{1cm} (2.11)

And, with the hypothesis of parameter independent from the temperature, we obtain:

$$\eta_t = \frac{c\gamma \pi R^2 l \Delta \theta_m}{P(t_0)}$$  \hspace{1cm} (2.12)

with $\Delta \theta_m$ the mean increase of temperature in the time $t_0$:

$$P = R_{\alpha} I^2 = k_r \rho \frac{I}{\pi R^2} I^2$$ the power transformed in heat due to Joule effect.

So, the Eq. (2.12) can be rewritten in the form of:

$$\eta_t = 1 - \frac{P_{\text{loss}}}{P} = 1 - \frac{2\pi Rl P_{\text{loss}}}{P}$$  \hspace{1cm} (2.13)

with $P_{\text{loss}}$ the power of the thermal losses for radiation and convection from the surface of the work-piece (excluding conduction losses at the contacts), [W]
and $p_{\text{loss}}$ the specific thermal power losses for radiation and convection from the surface of the work-piece (excluding conduction losses at the contacts), $[\text{W/m}^2]$.

We want to rewrite the thermal efficiency in this way, so we can make some considerations:

$$
\eta_t = \frac{1}{1 + \frac{2p_0 t_0}{c \gamma R \Delta \theta_m}}
$$

(2.14)

First, we want to minimize the time of process $t_0$ and also the specific superficial thermal losses $p_0$. In this way, we prefer fast heating. However there is the advantage due to integration in a few time.

Secondly, the effort is to maximize the denominator, but if we have a fixed work-piece, so without changing heat specific capacity $c$, density of material $\gamma$ and geometrical characteristic radius $R$ included, we can only change the jump of temperature $\Delta \theta_m$. This is not clear from the graph below, because is considered the same heating. Usually it is a requirement of the process but if we heat up more the work-piece, we will obtain a higher thermal efficiency, with all constant the other parameters, time included.

![Figure 2.10](image)

**Figure 2.10** Thermal efficiency as a function of bar diameter, for heating times 30–60–90–120–150–180–240 s
This approach to calculate the thermal efficiency is the same used in the code we will see in the next chapter; measuring the power in the work-piece, will be subtract the power losses on the surface, operation that will be integrated in all the time process, time step by time step.
3. Model and the code

“The greatest enemy of knowledge is not ignorance, it is illusion of knowledge”

Stephen Hawking English theoretical physicist

In this chapter, we will discuss about the methodology applied in order to get a ‘good’ discretization of the geometrical model. The steps of development in the code ‘Ansys EM’ that allows to analyze the problem and to get results, through the FEM software are: first building the model and second solving the simulation.

Discretization is the process of transferring continuous functions, models, variables, and equations into discrete counterparts. Between one point and another, there are some polynomial functions that will interpolate the results, showing continuous result at least for the solved variable, usually a potential, even if the solution is discretized. The result will be more suitable if the points chosen are closer, but to choose too many points leads to a long-time calculation; it depends from the accuracy required on the solution. Anyway, a good engineer must know where to concentrate the points and where not.

Simulations are useful after the theoretical approach to the design before prototyping in a laboratory, with all the economic effort of a research project. Simulating prevent not only to waste of resources, but combined with experimental activities that validate results, helps to get closer to the best solution in short time.

Figure 3.1 – Example of solution of the Poisson’s equation on a unit circle with a FEM software.
3.1 Finite Element Method

The solution of engineering problems is almost impossible with analytical methods because of the amount of problems to take into account at the same time:

- Usually geometry is more complex than a single basic shape. Usually engineers deal with one-dimension problems: when the problem is planar, the solution is harder but possible or at least imaginable; when the problem has more dimensions, it is better to change approach.

- Presence of multiple goal functions to minimize/maximize to get best result (i.e. best efficiency); with multiple design variables as input increase number of analytical equation to solve the problem.

- To deal with different physical problem, i.e. this work combines at the same time electromagnetic problem (EM) with thermal one (TH), increases the complexity of the solution for the relation between them.

- Given initial conditions, one solution EM depends from the previous TH, and it is difficult to estimate the solution due to the non-linear material property changing.

- Here a dynamic problem is not present, i.e. fluidodynamic where also chemical is considered in a common life situation like boiling water (thermal problem), where the liquid changes state into vapor (material property changing) and lift to the surface (dynamic).

In order to deal with this complex phenomenon, there are computer programs that help engineers to obtain solutions. Unfortunately, they cannot help taking decisions: they only do what they are told to.

This is one of the most difficult step to understand: FEM programs are really useful, but only for people who know how to use it, and it is not possible to learn how to use it without knowing the theory. A car can be driven by anyone, because if you press on the accelerator, you see and feel that the car is increasing its speed; the same thing happens for all the other controls, as a matter of fact you can see at the same time some result, in this case of moving. In the case of FEM programs, you can give some commands, but your results will be displayed later, when the simulation is finished after the solving of the equations. Also all the
right initial and boundary conditions, geometry problems and material must be imposed to the problem, otherwise the solution will show the result of another problem, not yours.

For this reason, we will discuss a lot on the settings and why we will take some decision in advance, to prevent errors or achieve better results after. Now we must know first what is required from the software to solve the problem, and then we will go into details.

3.1.1 PDE’s solution

For a real and precise model solution, numerical methods are often the only practicable alternative. The main objective of a numerical model is to solve a PDEs on a discrete set of points of the solution domain.

The continuous domain is divided into littler sub-domains, discretizing points into vertices of regular shapes. The distance between two adjacent vertices is the mesh size. Not only space, but also time, is subdivided into discrete intervals, and we call “time step” the interval between two consecutive times at which the solution is obtained. The PDE is approximated, or discretized, to obtain a system of algebraic equations, where the unknowns are the solution values at the discretization points. The system of algebraic equations can be solved with iterative techniques. After the discretization, it is necessary to check if the approximation is appropriate. For a successful solution, the numerical scheme must be stable, convergent and consistent. The scheme is stable if the solution stays bounded during the solution procedure; it is convergent if the numerical solution tends to the real solution as the mesh size and the “time step” tends to zero. The scheme is consistent if the truncation error (error introduced by the finite approximation of derivatives) tends to zero as well.

To have our solution, we only miss two steps for both electrical and thermal case:
- First, to make solvable the problem, we must impose the boundary condition.
- Secondly, to make the solution unique, we must impose also the initial conditions.
3.1.2 Boundary conditions

Indicating with Φ the unknown PDE’s function, not only it must fulfill the solution in the region R – the calculation domain, but also it must satisfy certain conditions on S, the boundary on R. The choice of the boundary conditions affects the final solution.

The boundary conditions that can be imposed are of three kinds:
- **Dirichlet** condition: it is assigned by fixing a determined value of the potential on a given boundary curve. In this way, the curve is characterized by a constant value of potential; thus, the equipotential lines result tangential to such a boundary:

\[ \Phi(r) = f(r) \quad \text{r on } S \]

- **Neumann** condition: the condition is assigned by fixing the normal derivative of the potential on a given boundary curve; the line crosses the boundary in a known way.

\[ \frac{\partial \Phi(r)}{\partial n} = g(r) \quad \text{r on } S \]

- **Mixed** boundary conditions (called “Robin”), combining the two previous cases.

There are two methods to solve PDE equations: the “Variational approach” and the “Weighted residual approach”. By applying them to a PDE system, they give the same set of equations to solve. In the Variational approach, a function to minimize is required, typically the energy of the system. The weighted residual approach starts instead from the differential equations of the system, which in a general case we can write as:

\[ L(\Phi) - f = 0 \]
Where $L$ is a linear operator (for example $\nabla^2$), $f$ is a known function (typically the source, referring to (3.10) and (3.11)), and $\Phi$ the unknown function. The approximation of exact solution can be obtained assuming that it varies according to a set of known functions, each of them multiplied by an unknown coefficient.

$$\Phi_n = \varphi_0 + \sum_{j=1}^{n} a_j \varphi_j$$

It is possible to determine these approximate function coefficients. Function $\varphi_0$ usually satisfies the Dirichlet boundary condition. Now the residual $R_n$ is introduced:

$$L(\Phi) - f = R_n$$

The residual function $R$ varies in the domain $\Omega$. The coefficients $a_j$ are evaluated by orthogonalization of residual $R_n$ to a certain set of $n$ weighting functions $w_i$, i.e., by zeroing the inner product:

$$\langle R_n, w_i \rangle = \int_{\Omega} R_n w_i \, d\Omega = 0 \quad i = 1, 2, \ldots, n$$

A possible choice is $w_i = \varphi_i$, i.e. weighting function is the same of approximating function. This approach is named Galerkin method. Substituting (3.15) and (3.16) in (3.17), is obtained a set of $n$ simultaneous linear algebraic equations that can be solved, considering boundary conditions.
3.2 Ansys Mechanical APDL

ANSYS© Mechanical APDL (Ansys parametric design language) is the software used for the analysis with use of parameters, it is fundamental for our investigation. It allows electro and thermal tasks, as our problem requires. The principal steps of developed script are illustrated in the flux diagram below:

Figure 3.2 – Flux diagram for the subdivision of different tasks for the software in several steps.
3.2.1 “!model” and Input parameter file

Actually the script will start with a file called “!model”, that recalls in order all the parts of the program. It is placed in the principal directory folder where there will be all the file created from Ansys and the user to run that simulations. There will be a subfolder named “Subprog” that will contain all the subprogram files. The software reads the file row by row; here is reported the script:

```
|************************************************|
|**** non-homogeneous conducting heating ****|
|************************************************|
finish
clear
/input,environment_emag,dat,Subprog
save,environment_emag,db,,all
!/eof
finish
clear
/input,environment_therm,dat,Subprog
save,environment_therm,db,,all
!/eof
/input,solve,dat,Subprog
save
```

This is a short script, used to make order in the structure of the simulation. It recalls only three other files, with the “/input” command, and then save them. After the command “input” there are other fields, separated by comma. The fields change command by command, but in the case of “/input” are in order: name of the file, extension, directory. Similar setting is for “save”, but all this information can be found on the software’s guide. All the words in a row after an exclamation point are commented, and Ansys does not read them. The command “finish” and ”/clear” are used to reset from previous settings, so the three executions are separated. The name of the files sounds already familiar: environment_emag, environment_therm, solve. The two environments set the simulation respectively for EM and TH, then the “solve” file will recall them many time as needed. This is done to avoid wastes, solving the problem without rebuild every step (of solving process) all the preprocessing steps again.

The “! input” file, where all the parameters are contained, will be not shown all together, but piece by piece when it will be necessary, because now there is a lot
of not understandable information. Not all the code will be reported, but only the useful parts.

### 3.2.2 The geometry

In a system of cylindrical coordinates, where $x$ is the radial component, $\varphi$ the azimuthal one and $z$ is the depth, a cylinder is defined with one face lying on the frontal plane $(x, \varphi)$ representing the billet. Outside of the cylinder is defined another volume region dedicated to the air, it is essential to consider the magnetic field and to bound the energy of the system.

![Figure 3.2 – Symmetric view of billet (1/8) with different temperature zones](image-url)

In the picture above, it is illustrated not the entire cylinder but only one eights. This is for the choice of using symmetries, to display better the behavior in different sections of the work-piece, but even more for the time computational saving. Therefore, the simulation time required, when the model had double equations at beginning, the time was more than double. It is understandable that comparing the same time computation for the whole model and the symmetric one (only because symmetries exists) it is possible to have a significant improvement on results accuracy. The red part is the required “hot zone”, and the orange the “cold one”, like in chapter 1.

The previous induction model has two different hot zones, and it was necessary to keep in count all of them in order to design the inductor. Here the choice to have only one hot zone is for investigation problem required, it is for more generality and for simplicity, that allowed to save time to spend in wider study. We will see
that this hypothesis is not restrictive, because this conduction and previous induction investigations can be compared anyway, due to the flexibility of the conduction system. Before showing some part of the geometry script, we need to know some useful parameter of input file)

```plaintext
!****************************************************
!*** 1.2.1 GEOMETRY - components ***
!****************************************************
!
! In geometry we define the parameter for the components, then their coordinates in the space

!**************************************************Workpiece**************************************************

h_wp=0.03 !Height of the workpiece diameter of 30mm
l_wp=0.474/2 !Lenght of the workpiece 20 cm

!************************************************************************Air-Box************************************************************************

h_air=h_wp !Height of the air, odd to symmetry
l_air=l_wp !Lenght of the air

!**************************************************************************1.2.2. GEOMETRY - coordinates ***********************

!**************************************************x-coordinates**************************************************

!Workpiece and air-box
x1=0
x2=h_wp/2
x3=h_air

!**************************************************y-coordinates**************************************************

!Workpiece and air-box centrated on 0
y1=0
y2=h_wp/2
y3=h_air

!**************************************************z-coordinates**************************************************

!Workpiece and air-box
z1=0
z_curr=0.0025
y_angle=75 !90°-y_angle=angle of contact area
z2=0.035 !35mm, half hot zone of 70mm
z3=l_wp

It is important to write the code in order, where also other programmer could work on it. The use of “!” to separate, indicate and comment is fundamental to read the code; most of all, it is the most convenient way because it is then
easier to find any possible mistakes. If the programmer writes fast without some boundaries and title, he must look over all the text to find a mistake, otherwise his attention is useful to avoid waste of time, especially when he is learning to code with the new software.

Now that all the parameters are clear, it is possible to look deeper at the second file diagram of the flowing chart previously presented, that is the geometry:

```
!*************************************************************************************************
!*** 02 GEOMETRY ***
!*************************************************************************************************

finish
/prep7
emunit,mks

!attention, cylindrical coordinate:
! x = radial component
! y = theta, azimuthal component
! z = longitudinal, depth component
csys,1

!*************************************************************************************************
!*** 2.1 PARTS, 2.2 NAME AREAS & colour ***
!*************************************************************************************************

!**************************
!*** Workpiece ***
!**************************
cylind,0,h_wp/2,0,z2,0,90
cylind,0,h_wp/2,z2,l_wp,0,90

!**************************
!*** Air ***
!**************************
rad1,rad2,z1,z2,y1,y2
cylind,h_wp/2,x3,0,l_wp,0,90
block,x1,x3,y1,y3,z1,z3

vsel,all
!vglue,all
vovlap,all

vsel,s,loc,x,x1,x2
vatt,200,1,200  !WP 12=red
/color,volume,12

vsel,all
vsel,u,mat,,100,400
vatt,400,1,400
vsel,s,mat,,400,499  !Air-box 4=cyan
/color,volume,4

allsel,all
vplot

save,geom_elec_db,db,,all
finish
```
There first part introduces the script with the place of the work, in this case “/prep7” means preprocessor, where in Ansys there are rules and command allowed. Special attention must be paid to the core, where the symmetric work-piece is built with the “cylinder” command. There are two cylinders in series for a temperature measuring reason from the hot to the cold zone, but they form the one eight of the billet. Outside there is the air, at first made by another cylinder, but then changed to a “block” because of the easier application of the electromagnetic boundary conditions. When two volumes intersect each other, to separate them we use the command “vovlap” (volume overlap); when they touch each other with a surface in common, the command used is “vglue”.

Sometimes we can find some commented commands that are useful when the programmer wants to make some changes and there are some ready working shortcut, he just needs to uncomment them. Then, the attribution means that all of the volume selected refers to the same number, so the two cylinders in series are recalled by the same name: in this case volume 200, denomination used as a convention to refer at work-piece, 400 for the air. This is the first file, while simulating the code, where the programmer sees something on the screen. This is because the first input file, where there are the parameters, does not build anything.

3.2.3 The mesh

In this paragraph, we will explain the discretization process, that is the passage from a continuous volume specified before to a set of point, where equations will be calculated. We will cut lines in various little segments, as much as necessary for a good approximation. Usually, while the programmer is writing the script, he begins with a poor set of points, to execute the software and correct coarse errors. Then, he fines the mesh where the gradients of interest are higher, or where the investigated phenomenon is concentrated. The computation here should takes more time. At last, when the execution is correct and the results are confirmed, he could improve the mesh removing points where they are not necessary or change the step of the divisions, with the purpose of making the following simulations faster. This is not the only way to achieve better solutions
or a lower computation time; it is possible also to affect the order of interpolation between the nodes. The default order of the software is the second, and to increase the speed of solution sometimes it is possible to reduce to one, so a linear interpolation, maybe only in some region where phenomena are flatter.

Ansys matches different problems into a choice of the “element type”: when the type of the problem is chosen, the software will build a net (the mesh) with nodes, and for each it solves the kind of equation for the specified problem.

There are a lot of type elements (more than 200), everyone optimized for the solution required. The difference among them are the shape element (triangular, squared, rectangular, hexagonal...), number of nodes per element, axisymmetric if present, dimensions (point, line, area, volume), degree of freedom: structural, thermal, fluid, electric, magnetic, or coupled-field, both for input quantities (loads) and outputs. The element type should be chosen such that the degrees of freedom are sufficient to characterize the model's response. Including unnecessary degrees of freedom increases the solution memory requirements and running time. Similarly, selecting element types with unnecessary features, such as using an element type with plastic capability in an elastic solution, also unnecessarily increases the analysis run time.

For the electromagnetic solution, we will choose one element type, and an other for the thermal, because even if it could have the same geometry the equation to
be solved is different. So, now, the consideration why will be built two different environments is clear: one electromagnetic and one thermal. Obviously, there will be an element type also for the air. The following piece of code from the file “3.Mesh” regards the selection of a line. A line is identified by its central point, so a selection and reselection of the same set are made, until one or more lines to divide are obtained, creating the point for the mesh.

\[
\begin{align*}
\text{lsl}, & s, \text{loc}, z, z2+\text{eps}, z3-\text{eps} \\
\text{lsel}, & r, \text{loc}, x, x1-\text{eps}, x2+\text{eps} \\
\text{lesize}, & \text{all,}, dz1, \text{ratio_wp}
\end{align*}
\]

The selection process could be easily done if in the geometry file we have spent more time naming every point starting from parameter, and then created lines connecting points with a name, and then areas from lines and at last volumes surrounded by areas. It is a longer process, but is easier in every step done later to select entities without referring by coordinates of the center but calling by name. Anyway, after the division of every line, (if a division is not fixed the software will give one automatically) the next step is the assignment of the element type. This is one of the most characteristic steps in Ansys, because in other software it is not present; usually the type of the analysis is chosen from the start, but this command gives the expert user more freedom and control over the model. An example of this assignment is:

\[
\begin{align*}
\text{et}, & 200, 236, 1, 2, 0 \quad \text{!workpiece} \\
\text{et}, & 400, 236, 0, 0 \quad \text{!air}
\end{align*}
\]

where the command is followed by the number of the volume built in geometry, then the element type (236) and other “keyopt” numbers separated by comma that indicate the properties. From the reference guide in Ansys we can find something about the element type 236, reporting only interesting steps:

“SOLID236 Element description

SOLID236 is a 3-D 20-node element capable of modeling electromagnetic fields. The element has magnetic and electric degrees of freedom. Magnetic degrees of freedom are based on the edge-flux formulation.

The edge-flux (AZ) degrees of freedoms are the line integrals of the magnetic vector potential along the element edges. They are defined at the
midside nodes only, and there are no magnetic degrees of freedom associated with the corner nodes. […]

In an electromagnetic analysis, the electric degree of freedom is the electric potential (VOLT) defined at each node. The element also has an option to perform an electromagnetic analysis with time-integrated electric potential. […]

The element is applicable to 3-D static, time-harmonic and time-transient electromagnetic analyses. The magnetic analysis option typically is used to model air, iron, nonferrous materials and permanent magnets. The analysis is driven by the current density applied as an element body load.

The electromagnetic analysis option is suitable for modeling solid (massive) conductors. The solid (massive) conductor may be voltage-driven or current-driven, as well as circuit-fed. […]”

The guide gives all the explanation regarding elements, even if it is written with a technical language with a lot of references. The suggestion is to spend time reading all the details to understand differences between similar element types. The assignment of one element type instead of another affects all the simulation in every field, determining the possibility to solve better, worse or even make unsolvable the model.

![Current](image)

**Figure 3.4** – Work-piece mesh, with increasing number of elements due to spacing ratio in the zone feed by the current.

From the following mesh picture of the work-piece we can see the different size of elements following mesh. It is not easy to estimate how far, and how much detail, to model may be gained by considering that the current falls off exponentially with depth. This behavior is due to the skin effect, that drop it to $1/e$ of its surface density, in a distance equal to one skin depth. “This is usually
adequately modeled by two or three first order finite elements, or one element of
the second or third order, per skin depth.”

In our model, the worst-case scenario is the frequency of 3kHz from the feeding
limitation, so the skin depth value is:

\[
\delta = \sqrt{\frac{\rho_{20^\circ C}}{\pi \cdot f \cdot \mu \cdot \mu_0}} = \sqrt{\frac{2.32 \cdot 10^{-7}}{\pi \cdot 3kHz \cdot 71 \cdot \mu_0}} = 0.525 \text{ mm}
\]

In our model, the element division on the outside transversal surface of the work-
piece was done following this aspect: half of the billet has a depth of half
diameter, so 15mm. That segment was divided in 30 parts, using a parameter like
dz1 in the example. A spacing ratio of 5 is applied, that means that from the last
to the first element there is a ratio of 5 in length. This allows to keep the number
of elements low, but also to increase their concentration under the surface. Due to
the second order element, only the 30-part division (15/30 = 0.500 mm) satisfied the requirement of “at least one element per skin depth”, but with this
allows more accuracy in the results, without wasting more time in computation.
The inner elements cannot be far away from each other, even if they are not
interested from electromagnetic phenomena, because in thermal diffusion problem
they will be the focus of this investigation, so a good accuracy is needed also
there.

### 3.2.4 Material property

We already said that material properties are one of the most crucial steps in
this work. That is why every model depends from which material properties are
used in the simulation and how they are built. First, it is necessary to know better
the work-piece material, the 42CrMo4.

The cylinder of 42CrMo4 steel.42xx steel is a family of SAE steel grades, as
specified by the Society of Automotive Engineers (SAE). Alloying elements
include chromium and molybdenum. They have an excellent strength to weight

---

1 Computer-Aided Design in Magnetics, D.A. Lowther P.P. Silvester, Springer Science
ratio, and are considerably stronger and harder than standard 1020 steel, but they are not easily welded (need pre and post weld thermal treatment to avoid cold cracking).

42CrMo4 alloy is a high-grade steel alloy containing, a part of Fe:

<table>
<thead>
<tr>
<th>Chemical element</th>
<th>Min % composition</th>
<th>Max % composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon (C)</td>
<td>0.38</td>
<td>0.45</td>
</tr>
<tr>
<td>Chromium (Cr)</td>
<td>0.90</td>
<td>1.20</td>
</tr>
<tr>
<td>Molybdenum (Mo)</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td>Silicon (Si)</td>
<td>0.17</td>
<td>0.37</td>
</tr>
<tr>
<td>Manganese (Mn)</td>
<td>0.50</td>
<td>0.80</td>
</tr>
<tr>
<td>Sulphur (S)</td>
<td>-</td>
<td>0.035</td>
</tr>
<tr>
<td>Phosphorus (P)</td>
<td>-</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table 3.1 – Chemical composition of 42CrMo4.

Many automotive companies and manufacturers use 42CrMo4 alloy to produce automotive parts by cold and hot forging. This material is commonly used to make components that are used for high stress applications, such as pinions, drive rods, crankshafts, springs and gears. Other applications for 42CrMo4 steel are also structural tubing, tool holders, stem assemblies, connection rods and locomotive traction. The analysis will be done on most influential characteristic quantities, considering the variation with the temperature, illustrated in the pictures below. All the material properties are taken from the previous study, to simulate the same material not only in its geometrical quantities, but also in its behavior. The first two regard the EM analysis, the last two regard the TH one:

Relative permeability decreases until it reaches 760°C (Curie point), where the steel is no longer magnetic.
The electrical resistivity increases with temperature, but not linearly. The growth is faster between 0 and 800 °C, and then slower, between 800 and 1200 °C. Resistivity at 1200 °C is five times higher the resistivity at room temperature.

![Graph](Figure 3.5 – Relative magnetic permeability of 42CrMo4)

![Graph](Figure 3.6 – Electrical resistivity of 42CrMo4)
The thermal conductivity $\lambda$ of 42CrMo4 measures its ability to transmit heat. It will be a very important property to heat the core point, unreachable with an effective and specific electrical heating, but only for conduction from outer hotter points. It is an advantage that the conduction is higher when the material is hot, because it means that the time constant of diffusion, with the same amount of heat, is lower compared to cold condition. With a lower time, it is clear from the thermal efficiency (2.14) that this allows better efficiency for thermal processes.

![Figure 3.7 – Thermal conductivity of 42CrMo4](image)

The dependence with temperature of specific heat capacity will be illustrated in the next page. It grows a little, but there is a high peak around the Curie temperature, five time higher than the specific heat at 20°C. That is because a lot of energy is absorbed in destroying the magnetic domains while heating, and also metallurgic structure changes.
In the assigning of material properties, the \textit{mp} command is used, the independent variable (x axis on the graph) and after that the variable affected (y axis on the graph) that in the example case is the relative magnetic permeability. Differently from lower dimension model, in a 3D one all the three components of a quantity must be specified, so in the script there will be \textit{murx}, \textit{mury}, \textit{murz}. A good rule, giving the characteristic point by point, is to give more points where the curve changes quickly its derivate, in the example of the thermal capacity an accumulation of values around the Curie’s temperature can be seen.

!relative permeability of the Work-piece

\begin{verbatim}
mp
temp
mp
temp, 1, 0, 100, 200, 300, 400, 500,
mp
temp, 7, 600, 700, 750, 2000
mp
data, murx, 200, 1, 72.0, 71.0, 67.0, 60.0, 52.0, 40.0
mp
data, murx, 200, 7, 26.0, 10.0, 1.0, 1.0
\end{verbatim}

\textbf{Figure 3.8} – Specific heat capacity of 42CrMo4
!thermal capacity J/(kg*K)

mptemp
mptemp,1, 20, 200, 300, 400, 450, 500
mptemp,7, 550, 600, 650, 700, 710, 720
mptemp,13, 730, 740, 750, 755, 757, 760
mptemp,19, 770, 780, 785, 790, 795
mptemp,24, 800, 810, 820, 900, 1000, 1100
mptemp,30, 1200, 2000

mpdata,c,200,1, 470, 500, 550, 575, 600, 650
mpdata,c,200,7, 700, 760, 850, 910, 930, 960
mpdata,c,200,13, 1000, 1150, 1300, 1600, 2000, 2400
mpdata,c,200,19, 2500, 2250, 1700, 1400, 900
mpdata,c,200,24, 900, 940, 970, 1000, 1050, 1100
mpdata,c,200,30, 530, 530

It is important to not forget to assign also the electrical material properties also to
the air element, otherwise the EM simulation will be invalid. For thermal
properties of the air, other considerations in the paragraph of thermal loads will be
given.

### 3.2.5 Electromagnetic loads

Applying electromagnetic loads is where the project takes another path
compared to the induction heating, because the work-piece is the same in
terms of geometry and in properties, though it is a 3D model instead a 2D axisymmetric
one. In this paragraph code, there is the first written explanation of the previous
choices, the application of the current in a parametric area and next the behavior
in the transversal plane of it and all the thermal phenomena. Again, the choice of
the element type is fundamental to set the right loads in the easier way, and to
establish a set of rules different for each one. From the reference guide, there is
the useful command to apply EM loads:

```
"SOLID236 Element description
The type of units (MKS or user defined) is specified via
the EMUNIT command. EMUNIT also determines the value of MUZRO
and EPZRO. The EMUNIT defaults are MKS units and MUZRO = 4\pi10^{-7}
Henry/meter and EPZRO = 8.854 x 10^{-12} Farad/meter. In addition to
MUZRO and EPZRO, orthotropic relative permeability and permittivity is
available and is specified through the MURX, MURY, and MURZ and
PERX, PERY, PERZ material options, respectively. Orthotropic resistivity
is specified through RSVX, RSVY, and RSVZ material property labels.

[...]
```
Nonlinear magnetic B-H properties are entered via the TB command, as described in Material Models in the Material Reference. Nonlinear orthotropic magnetic properties may be specified with a combination of a B-H curve and linear relative permeability. The B-H curve will be used in each element coordinate direction where a zero value of relative permeability is specified. Only one B-H curve may be specified per material. [...] 

Nodal loads are defined via the D and F commands. For edge-based analyses, the edge-flux constraint is applied to the node via the D command with Lab = AZ. Flux-parallel boundary conditions are prescribed by setting AZ to zero. No AZ constraint is required to set flux-normal boundary conditions. [...] 

For massive conductors (KEYOPT(1) = 1), Lab = VOLT is valid with the D command and VALUE defines the electric potential. Note that electric potential is time-integrated if KEYOPT(2) = 2. With the F command, Lab = AMPS and VALUE corresponds to the total current. [...] 

The temperature (for material property evaluation only) body loads may be input based on their value at the element's nodes or as a single element value [BF, BFE]. In general, unspecified nodal values of temperatures default to the uniform value specified via the BFUNIF or TUNIF commands. [...] 

Here it is reported in technical language, what can be done to apply loads, after the material properties. Indeed, there are many parameters that can be specified in terms of magnetic permeability, electrical resistivity or B-H curve. But now, the focus is on the following part, the imposition of loads. D or F commands could be used, depending from the load. For example, current will be applied using F command and voltage D command. The current will be applied selecting an area of nodes on the external surface over the work-piece. This doesn’t consider the non-homogeneous distribution of the current in the contact surface area, but it is a problem that can be studied later, because it introduces other objects, that modify the problem. It should correspond for symmetry, another equal area at the opposite side of the transversal side of the billet, but it is not possible because the model is cut in half in that direction. It is possible to make it work, but we have to assign an imposition of same electric potential to all the cut surface, because it is easier to consider the mirror flowing of the current
form the other equivalent side. In specific, our case is assigned $V = 0$ only to have a reference, but any potential is good when not connected to ground. Imposing ground potential is avoided any misunderstanding. Having all the “floor” (xz plane in Cartesian coordinate system) of same potential, the current is free to flow where it is most convenient, so it has the right physical behavior. All other possible configurations are wrong, because the symmetry or the physical behavior of current is not considered.

It is not clear from the picture that the area where the current is imposed is circular shaped, touching the surface of the billet. Also, the measure parameter given as input in the software reminds that “y_angle” is not a planar measure.

Once imposed the loads, this paragraph needs also the boundary conditions, as part of imposition quantities to make the problem solvable and unique. It is possible to subdivide boundary conditions in energy boundaries and electromagnetic boundaries. The first are boundaries imposed to confine all the energy used inside the model, so there will be imposition on the external shield of air of magnetic flux parallel. External walls are the left one, the upper one and the last in z direction. Figure 3.10 illustrates the imposition of bounded energy on these plane made of air with numbers 1,2,3:

**Figure 3.9** – EM imposition of loads on work-piece in terms of current and voltage.
In the case of element 236, this type of constrain is imposed with the command $AZ = 0$. The condition is not imposed on $B$ but on $A$ that is magnetic vector potential (from the definition $A = \nabla \times A$), because the formulation used to solve the problem finds $A$ as solution of equations, and then the magnetic field consequently. The electromagnetic boundary conditions are used for symmetric models like this, and impose the condition for the magnetic field on the cut planes. We have three planes where to apply BCs, because the representation is only one eight of the total. The programmer must think at the behavior of physical quantities, in this case current and related magnetic field. The current flows from the top to the ground, where the imposed potential is equal for each point, the magnetic field produced can be seen from the right hand’s law. The affected walls, where this magnetic field crosses the border with normal component are the front and the side, with number 5 and 6. This imposition does not require any command because, for element 236, the condition of normal flux is the automatic one. Another behavior has the bottom air surface, that one is on the same plane of the imposition of voltage on the work-piece. The magnetic field at that crossing
must be parallel to the surface, because of the normal direction of the current, that creates circular lines parallel to that plane. Due to this reason, plane 6 has the imposition of \( AZ = 0 \). Now we will report some code of the file “5.load_emag”:

```plaintext
csys,0
  nsel,s,loc,y,y1-eps,y1+eps
d,all,az,0,0
csys,1
  esel,s,mat,,400
  nsel,r,ext
  nsel,u,loc,x,x2,x2
  nsel,u,loc,z,z1,z1
d,all,az,0,0

!current on top
csys,1
  esel,s,mat,,200
  nsel,r,ext
  nsel,r,loc,z,z1,z_curr
  nsel,r,loc,x,x2,x2
  nsel,r,loc,y,y_angle1,y_angle2
  cp,next,volt,all
  *get,nmin,node,0,num,min
  f,nmin,amps,i_re/4

!volt on bottom
  esel,s,mat,,200
  nsel,r,ext
  nsel,r,loc,y,0,1
  cp,next,volt,all
d,all,volt,0,0
```

In order to select the current area nodes, we used a cylindrical coordinate system command. The applied current is not the total one, but only one fourth; this is an important consideration because the model is divided in eight pieces, but the surface where current is applied is placed over four of them, so we have a parallel connection. The other four remaining symmetric pieces are connected in parallel between them, and in series with the piece under the current contact.

After the selection of interest nodes, if it is an application of current or voltage, the user has to couple the selected nodes with a degree of freedom, in our example volt.
3.2.6 Thermal loads

Thermal environment is developed with the same geometry, mesh and material properties of electromagnetic one. The two environments are different because of the geometry and the element mesh, in this case element type 90 that is a solid 3D, used for thermal models. In this case geometry changes from EM simulation because air is a useless material: in thermal simulation, the work-piece is built without many nodes of a material that does not interact with simulation. Instead of building and deleting every time step the air material, with the creation of two saved environments, it is necessary only to load the proper environment and solve it. This process of doubling the files is worth it because the time computation to calculate solutions of equations is bigger than creating, meshing and solving for each time step, and then deleting it.

The principal thermal loads are radiation of the surface and convection. As initial condition, the temperature is fixed at 20°C, like the ambient one. Again, we present some code to understand the differences between EM and TH models:

```
esel,s,mat,,200
nsle
nsel,r,ext
nsel,r,loc,z,z3-eps,z3+eps
nsel,a,loc,x,x2-eps,x2+eps
sf,all,conv,h,amb_temp    ! convection load

esel,s,mat,,200
nsle
nsel,r,ext
nsel,r,loc,z,z3-eps,z3+eps
nsel,a,loc,x,x2-eps,x2+eps
sf,all,rdsf,rad_emis,1    !irradiation
nsel,all

allsel
toffst,273
tunif,amb_temp

stef,5.67e-8
radopt,1,0,1.0e-5,0,10000,0.9  !!!!!
SPCTEMP,1,20.0 !1-enclosure,20.0-outer temperature
HEMIOPT,10 ! (def, how many rays)
vfopt,new,,,,bina
v2dopt,1,,,,
rsurf
vfopt,new,vfmatrix,,,,bina
allsel,all
```
We must remember that most of the commands can be used in different position and order, but it is difficult to find and modify them, if they are not at their place. For this reason, the order and the subdivision in files is fundamental for the debugging phase.

With these considerations, the discretization part called “preprocessing” in Ansys, is finished. The last part of the model and the code is the solution, explained in the next chapter.
4. Solution and first results

“If you want something you've never had, you must be willing to do something you've never done.”

Thomas Jefferson – principal author of the Declaration of Independence and third President of the United States

In this chapter, we analyze how to set the software to get a solution, and the results of this simulation. This is not the first model, but one already developed and improved, so all the “how to code” steps are not present. The steps are: find a harmonic electromagnetic solution and then a transient thermal one, make a loop between the two simulations, and then close everything inside a loop that changes parameters. There are five nested loops, with estimated time simulation of one week. If the development of the code follows the order and the rules, there are good possibilities to achieve results in short time. After some code of the script, we will present the results of this solution.

4.1 Internal cycle: EM and TH

The two solutions have a different purpose: the electromagnetic is harmonic, so it is not dependent by time and produces as a result the file of power heating sources. This list reports the specific power calculated for each little element of the mesh in the work-piece. The thermal solution takes as input this file, and load it into a transient simulation, where the initial condition is loaded from the previous step. Then the specific power losses are integrated in time in different geometrical position and magnitude, considering the material temperature dependence, causing an increasing of temperature at the end of time step. Each temperature is written in a file, loaded as input in electromagnetic simulation, with purpose of changing material properties. This goes on until the exit condition from the loop is satisfied: i.e. time of 100s, temperature in hottest point of 1200°C, max difference of temperature between two points of 50°C.

The sum of electromagnetic and thermal solution is called in the script
“internal cycle” because it solves completely only one value for the four fixed parameters. To change them there will be the external cycle. To do the count for the internal cycle, we need a variable, called “iiii”. Its role is starting from one and add one to itself each complete internal cycle. The variable “zeit” has a different behavior, it is used to memorize the time, useful when the time step is not constant. Interesting variables are also the fixed parameters, in this case current and frequency imposed; the others are explained further. The other two geometrical parameters are imposed in the input file and the cycle is not developed also for them, otherwise the software would take at least a week to finish all the simulations. It is faster to get better results if some results are analyzed for each sub-simulation. Then we have to change the geometrical parameter in a smart way, otherwise the field of investigation is too big and the percentage of satisfactory results will be low.

The following code is for the internal cycle:

```plaintext
!electric
:beginning
/clear
/filename,ZF,1
resume,environment_emag,db
/input,iiii,dat
    /assign,rst,ZF,rmg
/prep7
    *if,iiii,eq,1,then
        tunif,amb_temp
    *else
        ldread,temp,last,,,,,rth
    *endif
/solu
csys,0
antype,harmic
harfrq,freq
allsel,all
solve
finish
/post1
esel,all
esel,s,mat,,200
eplot
POWERH
power=PAVG
```
In this very short way, all the variables used to make the cycle work are omitted, but the basic commands are present. First, the “:beginning” means that at the end of the thermal cycle, if the solution is not satisfied, the loop starts again from that point. Then is resumed the EM environment, and coupled with a electromagnetic problem with the assignment of .rst to the .rmg file. The use of “ZF” name is only to underline that the title or the name of the file opened at launch of the software is not bonded, but must be the same when different simulations are coupled. Then, there is an “if” statement: if it is the first cycle, the temperature is uniform everywhere, otherwise it will load from a file the previous temperatures. Once chosen the harmonic type of simulation with the frequency (parameter), the command “solve” will work. After the solution, we must save the power computed on each node, this is a task for the command “powerh” that prints also on the screen the total joule power on the model. Obviously, to have the right evaluation of power, we have to multiply that number eight times, which is the number of symmetries on the model.

In the internal cycle, there is also the thermal part; we report here only the principal steps:

```
!thermal
/clear
/FILNAME,ZF,1
.resume,environment_therm.db
/input,iiii,dat
   /assign,rst,ZF,rth
   /solv
   antype,trans
esel,s,mat,,200
ldread,hgen,,,,2,,rmg
esel,s,mat,,200
nsle
*if,iiii,eq,1,then
   ic,all,temp,20
*else
   ldread,temp,last,,,,2,,rth ! 2 - applied temperature as initial conditions to the workpiece
*endif
esel,all
nsel,all
```
neqit, 1000
runtime = input_data_transient(iiii, 3)
timestep = input_data_transient(iiii, 2)
time, timestep
zeit = zeit + timestep

cnvto1, heat, 5, .9
solve
finish

/post1
sucr, ext, infc, , x2
susel, s, ext
sumap, tf, tf, sum
/go
supl, ext, tf, 0
sueval, sum, tf, sum
sueval, losses, tf, intg

!-----------------energy---------------------------
energy = energy + power * timestep
!integral for the energy given
en_lost = en_lost + losses * timestep
!integral for the energy losses

!-----------------------------------------------
/input, post_out, dat, Subprog

iiii = iiii + 1

*if, Temp1, ge, 1200, then
*go,: thermal_while_exit
*endif

*if, iiii, lt, nstep,: beginning
:thermal_while_exit

As before, in thermal part everything is canceled, and all the parameters required (not reported) are loaded. The simulation is associated to an .rth file to couple the thermal solution, where a transient type is chosen. As a matter of fact, after the load of heat power sources from the EM solution and the previous temperatures, the choice of time represents an interesting part. The script gets the time step from an external file, “input_data_transient”, where it is possible to set all the time steps that the transient simulation will do. It should be interesting the development of adaptive time step in function of the variation of temperature in some points to save time, at the same way done with mesh.

In the post processing part, the thermal losses are calculated for that time step, evaluating the thermal flux going outside from the external billet surface. There is a little error in this consideration, because the circular frontal faces of the
cylinder are not considered, but their area is really small compared to the billet surface. Then we should remember that the conduction losses due to the contacts are not calculated, which influence the thermal efficiency. In the energy section, the thermal losses and the energy given of this cycle to the previous steps are added, in this way the thermal efficiency can be calculated at end.

A “post_out” file is necessary to save pictures of temperature solutions and register temperatures in some specific points, useful for the data analysis. Further information about these points will be given in the next paragraphs. The evaluation condition is that the hottest point called “Temp1” reaches 1200°C. If the condition is satisfied the loop is over, if not, the cycle begins again starting from the EM simulation.

4.2 External cycle: current and frequency

The external cycle is built to have a solution for each couple of investigation current and frequency. Investigation means that there are no other accessible studies on this application yet, so it is not a simple parameter study, but is required to find adequate ranges for parameter. There are other similar studies with other technologies, and experience from colleagues at ETP allowed the first hypothesis, on which could have been apply the field of investigation. First, it was decided to modify externally the cycle geometrical parameter, because each choice influences more the model than electrical input, so each should be evaluated case by case. The range was set between 1 – 4kA for the current and 0 – 3kHz for the frequency. Then comes the division, taking enough points inside these ranges, but not so much to keep a reasonable number of simulations. We set the division in current of 0.5kA and 0 – 50 – 1000 – 1500 – 2000 – 2500 – 3000Hz for frequency. This subdivision is changed with time and other two parameters, getting a final field investigation of different values. In the input file are defined:

```
dim_curr=4
*DIM,current,ARRAY,dim_curr,1
  current (1,1)=4500
  current (2,1)=4000
  current (3,1)=3500
```
current (4,1)=3000

\[ \text{dim}_\text{freq}=7 \]
\[ \text{DIM, frequencies, ARRAY, dim}_\text{freq}, 1 \quad \text{!parameter kkk} \]
\[ \text{frequencies (1,1)=3000} \]
\[ \text{frequencies (2,1)=2500} \]
\[ \text{frequencies (3,1)=2000} \]
\[ \text{frequencies (4,1)=1500} \]
\[ \text{frequencies (5,1)=1000} \]
\[ \text{frequencies (6,1)=50} \]
\[ \text{frequencies (7,1)=0.01} \]

These are two arrays containing the values for the investigation. The reference name in the external loop of the solution file is “jjj” for current and “kkk” for frequency. If the assignment of name variable remembers the variable contained or its purpose, it is easier to remember it; for example, all loop variables are called with a repetition of the same letter. The seventh value for frequency should be 0Hz, because that means DC, but the software doesn’t know the value zero. With good approximation, we can assume that 0.01Hz is the same as 0Hz in this study. The static analysis should be loaded, that is easier and faster, but also the structure has to change implementing an “if cycle” and looking from an engineering point of view, the easiest way to develop this is to approximate the static analysis with a very low harmonic one.

```
!------------------------------------------------------
!------ external cycle frequency and current
!------------------------------------------------------
:kkk_start
:jjj_start

!initialize

!------
!internal cycle

jjj=jjj+1
*if, jjj, le, dim_curr,:jjj_start       !current
*endif

jjj=1
kkk=kkk+1
*if, kkk, le, dim_freq,:kkk_start     !frequency
```
Again, we present only the most important commands; in order to focus on the main structure, we avoided all the initialization procedures at start and saving and updating commands, that use a bigger amount of line commands. The two loops are one inside the other, and inside there is the internal loop. The conditions pick one parameter per loop from the correspondent array and then change it next loop. For example, for the last set, the first simulation is \( j j j = 1 \) and \( k k k = 1 \), so the selected couple are current \( = 4.5kA \) and frequency \( = 3kHz \). The next simulation, the parameters are \((2,1)\) so values of \((4kA,3Hz)\) and so on, for a total of 28 combinations.

### 4.3 First parametric study

It is interesting to know which configuration in terms of current, frequency and area of contact is the best to achieve reliable results. In this first parameter study, it is enough to get results to understand the behavior of the parameters. For this reason, a Pareto front was developed, illustrated in the next page. Then data analysis will come and adjustments of the process.

To judge if a result of a simulation is better than another, we consider the first parameter \( \Delta T_1 \), that is the difference between the point on the surface under the contacts and the core point on the same cross section. This is considered the most influencing parameter as regards others, because of the high difference of temperature between points, and because the heat sources are concentrated on the external layer, so the core point is heated only by means of thermal conduction. \( \Delta T_2 \) is less important, because the uniformity of temperature on the cross section is reached in shorter time. \( \Delta T_3 \) is the less important parameter for the fact that the current flows on the external ring in AC, so the hottest points are on the surface, and there will be less difference of temperature between them. Area parameters vary between values from \( 5^\circ \) to \( 50^\circ \) for the angle related to the arc of circumference of cross section, and from \( 2 \) to \( 5 \) mm.
The graph shows results of simulations of different area configurations, considering the simulation as finished when the hottest point $T_1$ reaches 1250°C. The various configurations are indicated with different shapes and colors. Due to the high information density, it is not possible to divide also for current and frequency. A better representation, but only for the best area configuration, is reported in figure 4.2, where it is possible to see that with different colors. Without analyzing information on the graph, it is possible to see that the dominating solution is the one with grey dots, the configuration 10_5. The first number recalls the angle of azimuthal direction, the second is for half of the depth in longitudinal direction. So, the best configuration here dominates only with geometrical condition, as a matter of facts any value of current and frequency does not affect result, but only finds another point on that curve. To dominate another curve means that each solution represented with a different shape and color, is lower in the two directions compared to the others. Usually, there are

![Figure 4.1 Pareto results for first parameter study](image-url)
many solutions and the designer chooses one of them; here, we must make a
choice between different sets of solutions. Again, the decision here is easy,
because the grey curve dominates every other area configuration. In figure 4.2, it
is possible to distinguish between different current applications, and high
frequency has always the shortest time of the set. For example, in the set of red
dots of 4.5kA, the point that has a $\Delta T_1$ of 650°C has the highest frequency
($3kHz$), the second of 550°C has the second one ($2.5kHz$) and so on. The curve
that is formed is energy related, because temperature is proportional to the power
supplied with integration of time, that is energy. The choice to heat up the work-
piece until the hottest point reaches 1250°C degree was done to guarantee the
same judgment condition to each simulation, and it worked. The results are not so
much satisfying because we want a temperature uniformity in the cross section;
also long-time graphs show that is not possible to go under 150°C, that makes
difference between hot and cold zone, and it is unacceptable. Anyway, we will see
in the next paragraph if it is enough to satisfy the required conditions.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.2.png}
\caption{Figure 4.2 Best configuration for first parameter study.}
\end{figure}
4.4 Analysis of results

Lot of corrections were necessary to run successfully our simulation, due to the high knowledge of the programming language. The main task was to write and read changing parameters and results in external text files, to control the simulation while is running, because the interface uses commands from different languages (there will be an in-depth analysis in appendix). Next images show the best results configuration of parameters, 10_5 at 3kHz, 4kA.

4.4.1 Distribution of current

As the best result of EM simulation, we show the results in terms of distribution of current in figure 4.1, then we will make several observations.

The current is concentrated on the external ring of the billet, flowing from the contact area to the opposite one. The first consideration is about the skin effect, that affects the behavior of the current pushing it to the surface of the conductor. This should be negative because only the external ring is heated, but it has a general effect of increasing the value of resistance. We must remember that, heating a steel billet with contact on the cross section means a shorter path respect the front-end configuration. If in that way the current needed was high, due to the low resistance, here it is even more, because the path is one order of magnitude shorter. The effect seems to be like induction heating, where in a thin layer is...
concentrated most of the current, but it is different. If we look at power sources concentration, we see a high concentration of power right under the contact area, where the surface is lower. When we go far from this surface, the current flows in a bigger ring because the only constrain is the skin depth, but the surface is large, and the current squeezes and dissolves its concentration. Due to this, at maximum distance from the contacts there will be the lower concentration of power, but already a little far from them the power is not enough to rise the temperature in a satisfying way to be considered a good simulation.

4.4.2 Distribution of temperature

The results of best EM simulation show a not so good behavior of the current with a high concentrated power heating sources. Now, we see how it reflects into TH simulation. Fig 4.4 illustrates the final distribution of temperature, when the hottest point reaches 1200°C.

![Figure 4.4 Distribution of temperature in the work-piece](image)

The temperature distribution does not satisfy the requirements. Hot zone is not the whole section with the prescribed width, but a little surface under the contacts. However, the transversal section has a higher temperature than the other deeper sections, even its gradient is higher: this is a good sign, due to the ring distribution of the current, because the heating can be strongly localized. The second good signal is the longitudinal heating grows most on the surface but, at a some distance from the central section, the temperature is more uniform along the cross section because of the thermal losses on the surface that slow the
progression and allow the thermal conduction. Resuming this first result in a general way, it is good to have a local heating and the uniformity of temperature in cross section after some distance from the contacts; nevertheless, the values of temperature are unacceptable for the high jump of temperature on the cross surface under contacts, due to the not uniform distribution of the current on the surface.

4.4.3 Further considerations

The comments previously made regarding the Pareto front are studied in-depth here. These graphs show some bad simulations, the ones that do not reach the temperature goal; nevertheless, they are useful to demonstrate that current and frequency do not affect the final difference of temperature, which we want to minimize. All the points are simulations stopped at the same time of 300 seconds.

![Graphs showing relation between max temperature reached versus max gradient](image)

**Figure 4.5** Relation between max temperature reached versus max gradient
The graphs above show those simulations that do not reach the goal temperature, and connect the reached temperature with the first indicator, that is jump of temperature $\Delta T_1$. Low frequency and low current are not useful to heat up the billet fast enough, but we already could know it, because direct resistance heating requires a lot of current due to the low resistance of the load. DC should be the solution to distribution problem, because the current flows also in the core point, but the resistance in that case is lower than AC, and without skin effect the results are the lowest point on previous graphs: the temperature does not grow.

The result from these graphs is that current and frequency are not useful to establish if a simulation is better than another, because at same time of heating, the slope that relates the reached temperature and the jump between surface and core is the same. This is a smart indicator that shows, without reaching the goal temperature, if a simulation is good or not, only for its slope. A low slope indicates that reaching an elevated temperature, the jump between surface and core is low. The lowest slope is the best simulation, recorded for 10.5, the same as pareto front analysis.

With these practical cases, the relation between current and temperature is underlined: punctually the last one depends proportionally to the specific heat power, and the current influences the power with magnitude of two. Between them, there are a lot of quantities that affect the relation; first of all, the geometry and the thermal losses, but an increase of current leads to a sensible increment of temperature. On the other hand, the frequency is not as good as current. A low frequency is required, otherwise without a skin effect the resistance is very low, but an increment of frequency, due to its squared dependence to power means that a big increase of frequency is required to affect the temperature at the same way as current does.

### 4.5 Energy problem

From previous data analysis, it is clear that a process that uses only non-homogeneous conduction heating is not acceptable, not only in terms of efficiency or details, but because it does not respect the first requirements of temperature distribution. For this reason, the analysis changed the focus, and this is not on the
already working code, not even to achieve better results: the energy required to
heat up the work-piece. Changing the point of view is possible to understand why
the behavior of temperature was far from the desired result.

Like we did before, the specific heat capacity is not considered variable with
time but we took a mean value, considering the initial and final temperatures of
the heating process. In this way, it is possible to obtain an easier formulation:

\[ E [J] = P(t) \cdot t = m \cdot c_p(T) \cdot \Delta T \]

The mass is calculated knowing the volume of the body and its density:

\[
m [kg] = \gamma \cdot V = \gamma \cdot (r^2 \cdot \pi) \cdot z = \\
= 7800 \cdot (0.015^2 \cdot \pi) \cdot 0.474 = \\
= 2.61 kg
\]

The specific heat capacity is considered a mean value of 710 \([J/(Kg\cdot^\circ C)]\) and the
\(\Delta T = 1200 - 20 = 1080^\circ C\). With these components, the work-piece, in order to
be heated in its totality, needs this energy:

\[
E [J] = m \cdot c_p(T) \cdot \Delta T = 2.61 \cdot 710 \cdot 1080 \equiv 2MJ
\]

The billet, heated up with cold and hot zone, requires less energy:

\[
E [J] = \left(m \cdot c_p(T)\right) \cdot \frac{1}{l} (\Delta T_{hot} \cdot 2l_{hot} + \Delta T_{cold} \cdot l_{cold}) = \\
= (2.61 \cdot 710) \cdot \frac{1}{0.474} (1080 \cdot 0.07 \cdot 2 + 880 \cdot 0.334 \equiv 1.75MJ
\]

This result shows clearly that there is an economic convenience to heat up only
the interested hot parts and leave cold the cold zones, because for each piece there
is a saving of 250kJ, that is not so much in terms of cost for energy, but producing
a lot of pieces, it becomes a concrete saving.
The question moves from the energy requirement to the comparison between this theoretic value and the one had with simulations. We took the higher value of mean power for this calculation, because it changes step in the simulation each time, multiplied for the time process.

\[ E \ [J] = P_{mean}(t) \cdot t \approx 3.5 kW \cdot 100 = \ 0.35 MJ \]

Comparing the two energies gives the conclusion:

\[ \frac{E_{\text{given}}}{E_{\text{required}}} = \frac{0.35}{1.75} = 0.20 \]

The simulation cannot work to obtain the required results, because we are giving an energy to the work-piece in that time that is 20% of the required. To have more power, it is not possible to rise again the current, nor the frequency. The only solution is to change approach, and this is what is done in the next chapter.
5. Process optimization

“Do not hurry, do not rest”

Johann Wolfgang von Goethe – German writer and statesman.

The previous approach, that is the use of only non-homogeneous heating to warm the billet, does not produce the desired effect; because of the low resistivity of the metallic load, that does not allow to generate enough power. The change of strategy considers the non-homogeneous heating good to shape the temperature locally, meaning that the work-piece should be already in the condition of cold zone. Therefore, we must split the heating process in two processes, which the first one has the following characteristics:

- Energy intensive: rising the temperature from ambient to cold one (850°C), about the 80% of the energy is required, from the results of chapter 4.
- Fast enough because if there are two processes in series, so the required process time increases (the production rate decreases and the investment is repaid in a longer time).
- Uniformity of temperature in all the work-piece, allowing the further forming process of non-homogeneous heating.

Various processes are already available to respect these requirements; in the electro-thermal field, there are induction heating and direct resistance heating.

![Circuit for resistance and induction heating.](image_url)

**Figure 5.1** Circuit for resistance and induction heating.
To be congruent with the theme of this work, and for the already developed approach with this technology, the first process that we will use is the direct resistance heating. If we make firstly the non-homogeneous heating, the gap between the hot zone and cold one will be canceled by the bigger amount of energy released by the first process. The order of the two processes is fundamental.

Theoretically, a two steps process should do a better work than a single one, we need to discuss at which temperature should finish one process and start the other. Neither with induction heating, nor with direct conduction heating, a system made of two processes in series could work at the same time with non-homogeneous conduction heating. This is due to the geometry of the heater, or for the presence of different electric fields that will cause the wrong behavior of current in double contacts configuration. So, the two processes must be in series, and we must consider also the thermal losses of the cold part, that will make the work-piece colder while the hot zone will be heated. Therefore, in this simpler case of investigation, we chose a temperature of 900°C. It is a little bit higher than the lowest temperature permitted of cold zone of 850°C, and it releases an energy of:

\[ E_{\text{lost}} = m \cdot c_p(T) \cdot \Delta T = 2.61 \cdot 710 \cdot (900 - 850) = 92.7 \text{kJ} \]

Considering that energy, the power is required to establish a time for the heating process that allow us to keep the temperature of cold zone higher than 850°C. There are two components, convection and radiation. For the experience of ETP’s members it was suggested to use values for still air convection coefficient of \( h_{\text{conv}} = \frac{W}{(m^2 \cdot K)} \) and steel emissivity of \( \varepsilon = 0.3 \). Here it is the estimation of the surface power losses, considering the external surface of the billet at 900°C:

\[ S = 2\pi \cdot r \cdot l = 2\pi \cdot 0.015 \cdot 0.474 = 0.045 \text{[m}^2] \]

\[ Q_{\text{conv}} = h_{\text{conv}} \cdot S \cdot \Delta T = 4 \cdot 0.045 \cdot (900 - 20) = 167.4 \text{[W]} \]
From these surfaces, power losses give a suggestion for time of the non-homogeneous heating process that should last:

\[ Q_{\text{emiss}} = \varepsilon \cdot \sigma_n \cdot S \cdot (T_1^4 - T_2^4) = \]
\[ = 0.3 \cdot 5.67 \cdot 0.045 \cdot \left( \left( \frac{1173}{100} \right)^4 - \left( \frac{293}{100} \right)^4 \right) = \]
\[ = 1444[W] \]

The suggested time for non-homogeneous heating process is about one minute, otherwise the surface temperature will be lower than 850°C. So, the choice to heat up 50°C more of required the work-piece is not wrong, but it will reflect on the thermal efficiency. If we will see that the process requires more time, we should rise also the offset temperature now set to +50°C, but we look for a low as possible time process.

In the next paragraphs, there is an in-depth development on the two processes; we try to optimize each of them, and to develop the code part required to make them work.

### 5.1 Direct heating conduction

It is not so difficult to apply a different heating on the code, but it requires to change electromagnetic environment that recall it, in the load script; everything else is still the same, solve file included. We made a minor modification to the input file to have another parameter to regulate the direct heating current and the non-homogeneous one separately. For the frequency parameter we will do consideration further. In this application of high current again, generators cannot satisfy any request from the user, because they are limited in current for the safety of its internal component. For this reason, the connection is directly made from the low tension of a transformer, that is a simpler but heavier electrical machine, and usually it can support stronger load in terms of current. As a matter of fact,
our transformer at ETP has the max available current of 4.5kA, so this is the new maximum value used for the current. However, the choice of using a transformer allows us to use only the industrial frequency of 50Hz. For the reason we explained before, a higher frequency is good to have a higher equivalent resistance and, with the same current, that means more power. Anyway, it leads to a not uniform distribution, flowing on the external part because of the skin effect. The aim of this heating is to reach a uniform temperature distribution at 900°C. It is already possible to make some considerations, but we will explain them after the results. The code of the load script is changed as follows:

```plaintext
!*** 05 homogeneous electric magnetic environment

csys,1
esel,s,mat,,400
nsle
nsel,r,ext
nsel,u,loc,x,x2,x2
d,all,az,0,0
esel,s,mat,,200
nsle
nsel,r,ext
nsel,u,loc,z,z1+eps,z3-eps
d,all,az,0,0

!current at end of wp flowing in z direction

esel,s,mat,,200,
eplot
csys,1
esel,s,mat,,200
nsle
nsel,r,ext
nsel,r,loc,z,z3-eps,z3+eps
cp,next,volt,all
*get,nmin,node,0,num,min
f,nmin,amps,i_direct/4

esel,s,mat,,200
nsle
nsel,r,ext
nsel,r,loc,z,z1-eps,z1+eps
cp,next,volt,all
d,all,volt,0,0

allsel,all
freq=2000
```
The code at first sight is very similar to the load seen previously, but it is just an appearance. It has a different selection of points on which the impositions are made. The boundary condition, as before, is made by a part that closes the energy inside the system and the second part that considers all the symmetries. After, there is the imposition of the current, called “i_direct” and divided by 4 even this time, and applied uniformly to the front face. In the bottom part, there is the ground connection, where the circuit is closed. Perfect contacts are considered, with a uniform distribution and so density of current that is difficult in reality, but it is the most general imposition.

5.1.1 Distribution of current and temperature

Only with this little modification, we can run a working simulation with a completely different behavior. In the solve file, the final temperature is changed: it is set at 900°C. In picture, the current is flowing not in an azimuthal direction, but uniformly distributed in a longitudinal one: this is a case of direct heating.

![Current distribution in direct conduction heating process.](image)

Figure 5.2 Current distribution in direct conduction heating process.

Figure 5.2 shows the behavior of the current at initial stage of heating: we
can see the skin effect, and the current forced to flow on an external skin layer. It seems the same problem of non-homogeneous conduction heating, but now the power sources are in the whole surface. Therefore, the power is much higher than before, and there is not some power concentration point, but the process is closer to get the uniformity also on temperature. Below, the figure illustrates the distribution of current in a direct way, the first at start and the second at a progressive stage.

![Figure 5.3 Current distribution in direct way at different stage.](image)

We can understand from the pictures that the power sources are more uniform while the billet rises its temperature. There are some specific characteristics helping us:

- Over 760°C, the Curie point, the steel is not magnetic anymore; that means that the skin effect has reduced, and the current is influenced mostly by other parameters.
- The geometry is uniform in longitudinal direction, so the current does not choose its path like in non-homogeneous heating, and the power sources will be uniform distributed.
- The electrical resistivity grows with temperature, so the current will
choose a more internal path, due to the previous higher temperature on the surface that has a higher resistance. This is possible only because the steel is not magnetic anymore.

- An effect to keep in mind is the already mentioned “volcano effect”: the surface layer is hotter when Curie point is overcome, but then it has thermal losses and fewer power sources as before, so the slope of temperature increasing is lower. In the opposite way, the internal part rises fast its temperature due to the conduction from the external layer, it does not have thermal losses and has power sources concentrated there. There is a time point where both zones have the same temperature, and from there the internal one grows more than the external. In a practical experiment, we have to pay attention to this, because the measured temperature outside could correspond to an already melt material internally, that could eruct hurting people or damaging equipment, or at least break the billet or change its material properties.

In our simulation case, the temperature to reach does not allow to generate an evident volcano effect. Figure 5.4 explains the core temperature and the surface one, measured in one of the simulations.

![Figure 5.4 Graph of core and surface temperature in a direct heating simulation.](image-url)
So, an AC heating is better than DC one, if we want the same uniform distribution of temperature in short time, because AC does produce more power with the same current, due to the higher equivalent resistance of the billet when it is still magnetic. Pictures below show the temperature at beginning of the heating and at the end, when the billet reaches 900°C.

![Temperature of billet](image1)

**Figure 5.5** Temperature of billet a) at beginning of the heating (20-70°C)  
b) at end of the heating (880-900°C)

### 5.1.2 Estimation of direct conduction heating parameters

Bessel functions are the solution of the problem, but in laboratory it is not an easy way to choose the best input parameters (current and frequency). There are some simplified formulas, but most of the time, experience drives these choices. To make it easier here, we want to develop some tables for practical purpose derived from these simplified formulas that allow the choice of input parameters as function of the equipment or requirements. Of course, these tables
will work properly only with this billet with these geometrical dimensions, material properties, in unreal conditions because are not considered some effect such skin effect of contacts, thermal losses on faces, perfect geometry of billet, and so on. Anyway, we decided to develop them because they will give indications, to understand if the chosen parameter for the heating of another billet could work or there are some coarse errors. There are the simplified formulas that connect parameters, and we want to check if these can simulate with a good approximation the time required for the heating. At last, some of the results are matched with simulations results, and we will see if it is worth it, because formulas do not consider thermal losses and material property changing.

All the tables contains comparison and relations between results of formulas and simulations. So, first of all, only one power result of simulation is needed, with its input parameters: frequency and current. Everything should be easier, because all the systems of thermal and electromagnetic equations are not required, but only the knowledge of relations between quantities. As said before, in a current driven problem, doubling the current means quadruplicating the power. In the same way, doubling the frequency means to multiply by $\sqrt{2}$ the power in a restricted field, because when frequency is 0 also the power is 0, but when the frequency tends to infinite, the power is again zero. Starting from these simple relations, we will see if they are enough to simulate this table. Time required is calculated by dividing the energy to give to the work-piece. This energy is not 1.75MJ anymore because it considers also the non-homogeneous energy, it is a bit less: 1.63MJ. The remaining energy to be used for the non-homogeneous process, is about 120kJ for two zones, the half for one like in our model. There are columns also for skin depth before and after Curie (b.C. and a.C.), and the calculation of coefficient m. In the table, we can see different colors:

- In yellow, there are the rows for which a complete simulation has been made, to make a comparison on resulting time.
- In red, the column of calculated and simulated time.
- In blue, the cell of power from which are calculated the other values.
  - In green, the cells referred to the frequency for the condition of true heating where $m \equiv 3.5$
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<th>1/8 Power</th>
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<th>Skin depth a.C.</th>
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<th>m a.C.</th>
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<td>[W]</td>
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Table 5.1 Table for the prediction time for direct conduction heating process.
The table demonstrates that time calculated, compared to time simulated, is closer, in an acceptable deviation range for 1.5kHz frequency. The error is proportional with the increase of the frequency, because it affects directly the skin depth where the sources of power are concentrated. In that case, the surface losses are bigger in the simulation, due to the higher temperature on the surface, and it influences the results with a higher time of simulation.

The behavior at frequencies higher than 1.5kHz is clear, and it increases the deviation, but these considerations are not true for lower frequencies. In fact, for true heating after Curie, the error in time increases, because the frequency is higher. On the other hand, for true heating before Curie at 150Hz the time simulated is one half than the expected one. The reason for this is the power induced in the work-piece is higher than expected, but the needed energy is always the same.

These low frequencies are not interesting for our process because there is not so much power, with limited input current, even if it is the double than expected, because we want to heat up the work-piece in a brief time. Where the time and so the efficiency are not the purpose, these frequencies could be considered. Therefore, the results in time of true heating frequency show only that these relations do not work for low frequencies, but there is a good approximation in our working field.

![Figure 5.6 Heating Power required versus frequency, for different currents](image-url)
Even if values inside a table are useful, graphs are more immediate, also for practical use. These graphs illustrate the previous table, with input of frequency and current, the calculated time and power.

**Figure 5.7** Heating time required versus frequency, for different currents.

Due to the relations, the interpolated points are part of the same curve that respects the relations. Power grows with the magnitude of one half in direct proportionality with frequency like in figure 5.6, instead time has a relation with frequency of inverse proportionality.

### 5.2 Non-homogeneous conduction heating

The direct conduction has stopped, and whole billet is uniformly heated at 900°C. Is it possible to manage some parameter, knowing the range of their work, to make possible to reach our goals? And if it is possible, there is a way to optimize this behavior in terms of speed of heating and uniformity of temperature?

These are almost the same questions made at the beginning, but the
conditions are different. The jump of temperature is lower and the task is local as we know this heating is good. One minute is the time to reach the goal, otherwise the surface temperature will drop more than the lowest temperature of the requirements for cold zone. At last and we know the behavior of the current and its concentration on the external part.

There is also a significant difference from the previous simulation: the steel is not magnetic anymore. Therefore, the current distribution, as for direct heating, will be more uniform, and will follow only resistivity and geometrical laws. The temperature is uniform at the beginning, so the resistivity does not influence the path of the current. We are dealing only with geometrical problems, that are not at all negligible: the case is very similar to DC imposition at low temperature. As a matter of fact, the current flows in each section, but concentrates the power sources where there is the bigger resistance, and that point is under the contacts due to the lower transversal section to the flow. This is not good at all for the power, which will be lower than previous, and less power means more time, but we do not have a lot of time.

![Figure 5.8 Distribution of current in transversal section after Curie in non-homogeneous heating.](image)
This last discovery about the power is a little bit annoying, because the feeling to be close to the achievement of goal was high, but a little further step is required. We have a hot point under the contacts, and the other does not warm so fast. It is neither possible to rise the current again, nor the frequency. We could draw the contact area wider, but the power will decrease due to the lower total resistance, or we can make it smaller with a higher power, but difficult to control because only the points under contact will warm up, with the risk of melting without reaching the uniformity results. Seems to not having any other possibilities, because dealing only with the parameter’s ranges seems to be not enough.

Yes, there is another way: if the hot points are close to the contacts, and it does not depend from the work-piece material properties, we can rotate them along azimuthally direction. This allows us to transfer a lot of energy not to only one point on the surface but to many, so they will not melt and the stability of the process is higher; at the same time, it is possible to heat wherever the user wants to. The connection between the transfer of heat and the position of contacts is strictly related. Obviously, the contact in opposite position must rotate alike.

This study does not go in-deep to look for the best velocity at which the contacts should rotate, but wants to give the opportunity to get closer as possible to the goals, respecting the field range of parameters. First, we implemented the missing code, then we show the results.

5.2.1 Implementation of the code

The code implementation starts with the loading of previous files from the other simulation; the requested files are “.rth” and “.rst”. The movements of contacts consists in the addition of a cycle that every loop changes the position of contacts. It is developed by means of a variable called “move”, and identified by position number 6 in the array variable that saves value to a file along the process. First, in the input files is created an array made of prefixed steps, that are repeated in loop, so the position nest to the last is the first. The contact moves only in an angle of less than 90°, depending by its size: if it is large 10°, the max movement is of 80°. The two parameters in this case are “dim_ang”, that specify how
movements, for example 10 times, and “angle” that indicate the dimension of it, for example 10°.

!movement of contacts in non homogeneous heating
!row defined by parameter "move" in thermal cycle

dim_ang=10
angle=10
*DIM,ang,ARRAY,dim_ang,2
*do,aa,1,dim_ang
ang (aa,1)=(90-angle)-((aa-1)*(90-2*angle)/dim_ang)
ang (aa,2)=ang (aa,1)+angle
*enddo

This in the input file is useful because each other recalled file has already in its saved memory this array that indicates, in azimuthally direction and in a cylindrical coordinate system, the initial and the final point of the contact. The imposition on the loads file is not reported due to the little modification to make: when the area of contact is selected, it is not going from 90° to the “y_angle” quantity but takes value from that array, knowing at which time step we are now, red in the move variable. Passed 10, this variable must be reset to 1 to restart the loop.

The electrical condition of symmetry is not correct, because there is not all the area of the contact that rotates, but only half, and the other half on the right side behaves like a mirror. In the same way, the contacts of ground move in the direction of visible moving contacts. For this reason, the contacts are not forced to do all the movements until plane zero, otherwise there will be a short circuit without power. Therefore, it is not possible to represent this kind of situation keeping a symmetric model like this, but we need to develop the fully 3D model to simulate the proper rotating system without any approximation.
5.2.2 Distribution of the current and temperature

Anyway, this is a conservative situation for simulation, because all the position after the first are in a lower power condition for the lower resistivity. We are simulating a worse heating process, that will be faster with adequate model with same conditions. It is interesting to underline the uniformity of current, in different position of the contacts, illustrated in the pictures in the next page. Below, there is the thermal process, where we can see how this new system of rotating electric contacts is working.

![Figure 5.9](image)

**Figure 5.9** Distribution of current in transversal section after Curie in non-homogeneous heating with rotating contacts.

The current follows the shorter path and it goes towards the ground potential, so in the second picture the flow of the current is more present. It is a simple problem, but we should find a solution or with already complete 3D model or with periodic symmetry: the contacts rotate in real world, but the movement is relative. In this way, we can rotate the billet, displaying how much contact
influence heating, keeping symmetries. A suggestion could be to work with a geometry of centered contact; to simulate a right behavior; the contact must be placed in the middle of half of the billet, like in a two-pole electric machine. If more pole is added, there is a current flowing problem, but this is not the task of this work. Below, we present the temperatures at the beginning of the process:

![Temperature Distribution](image)

**Figure 5.10** Distribution of temperature in transversal section at start of process

It is evident how the contacts rotate, because the hot point is where the temperature is higher, and the temperature is higher in the whole external surface. Now is displayed another set of rotating contacts temperature, but at the end of the process, so when the core point reaches 1150°C.
There is a slight difference of temperature in scale between them, but there is a good explanation of what this heating is doing. With all the ring at higher temperature, the central point of the billet (corner right down in pictures) receives the conduction contribution not only by the higher hot point, but from all the external surface. We have transformed the non-homogeneous conduction heating from a rude process that cannot even achieve requirements, to a smart process that has the possibility. This was possible only by setting the right conditions, after we studied the strengths and the weaknesses of the process.

Figure 5.11 Distribution of temperature in transversal section at end of heating in non-homogeneous heating with rotating contacts
5.3 Second parametric study

Before judging the results of the simulation, we made a second parameter study over different geometrical and electrical parameters. The first time, the dominating set of solution was the “10_5”, that means a small contact area and deep in longitudinal direction. Worse results were reached by wider and shorter configurations. The same approach is used also this time, and for a better comprehension of the problem, a little table is reported below; but first, we report the conversion formula to transform the angle unit to an arc length, and from the half of the arc and the half of the depth to the area:

\[
\text{arc length} = \frac{2\pi \cdot r}{360} \cdot \text{angle}[\text{mm}]
\]

\[
\text{Area} = z_{\text{curr}} \cdot \text{angle} \cdot 4[\text{mm}^2]
\]

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<th>(Z_{\text{curr}}[\text{mm}])</th>
<th>(Y_{\text{angle}}[^{\circ}])</th>
<th>Arc length[\text{mm}]</th>
<th>Area [\text{mm}^2]</th>
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\textbf{Table 5.2} Table representing all the configuration tested in previous parameter study.
Various combinations were tested, but the power given to the workpiece is strictly connected to the area, as we already demonstrated. Like the previous, it displays a Pareto front where different set of simulations were done, represented by different colors.

This time, the simulation was done on the availability of the constrains of frequency and current, to stay in some range areas:

- Lower than the previous, because the power now is diffused without risk of overshooting, due to the rotation of contacts $\leq 55\text{mm}^2$.
- Not so small for the stability and control of the local increase of the temperature on the surface $\geq 40\text{mm}^2$.

These considerations allowed to save a lot of time, giving more space to the working simulation, represented by points on the curve.

![Figure 5.12 Pareto front of second parameter study.](image)
This Pareto front is different from the precedent parameter study, in fact all the points belong to the same curve. There is not any more the parameters for the jump of temperature from external surface and the internal, because that problem was solved. Those are simulation that finish when the core point reaches the minimum temperature to be considered hot zone of 1150°C, all the other points of that surface have higher temperature.

There is not a winner or a loser, but there are different set of area configurations. These are available to make a choice further, for different simulating purposes.

We prefer to obtain the result in less time, so for this study the best configuration is 25_2: large of angle, and short in longitudinal direction. Maybe there are other better configurations, also considering the goal of the depth of contact, but this allows reaching the temperature of inner point faster than the others. Here it is reported the table of different simulated time, for which the 25_2 configuration finishes:

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Table 5.3 Table representing the finish time for 25_2 configuration, with current and frequency as parameters.
This table is another pleasant result, because the frequency is not so influential on the time heating, when the temperature is high. Supplying the work-piece in the direct conduction heating with a current at industrial frequency has a result not so different than supplying the same billet with an higher frequency.
6. Comparison between conduction and induction heating

“There will always be another opportunity, another friend, another love, a new force. For every end there is a new beginning”
Antoine de Saint-Exupéry – French writer and aviator.

The last chapter gives a verdict over the investigation carried out and compares the two heating methods.

6.1 Results of the investigation

The parametric study revealed an important improvement for this alternative process of non-homogeneous conduction heating. Starting from the approach, initial results are complete different from the last ones, because the condition of heating has changed. It was proved that this type of heating is not good for massive heating but only for shape the temperature distribution, so it needs another process to heat the work-piece till an offset temperature and then it raises the temperature in desired zones. It is a very local heat treatment due to the geometrical shapes, that concentrate the power sources under the contact’s surface. Most of requirements are reached, in particular:

- hot and cold zone temperatures for all the volume, with temperature from 850°C to over 1300°C
- uniformity of temperature in the transversal surface, with a system of rotating contacts allowing the control of the temperature and the power on the surface of the billet
- optimization of the time of the process to be compared with induction heating, in the fields of 100s

It has to be considered that the process involved here are two and not only “one shoot” like induction heating, so the time and the thermal efficiency will be
affected by the intrinsic impossibility to apply both heating at the same time.

This part of research is not finished yet, because the uniformity of temperature in longitudinal direction is a task that was not possible to improve in this work. Is displayed now the temperature profile in longitudinal direction with blue color, parameter not studied. In red there is the temperature profile required.

Figure 4.1 Temperature profile in longitudinal direction following  
   a) external top surface  b) external lateral surface
From the graphs, it results that the longitudinal requirement was not satisfied, because there is part of hot zone that is not warm up enough. Anyway the temperature profile is interesting because is a recall of one of the positive signals caught from first parameter study: the high temperature gradient. The x axis is measured in cm and the drop of temperature is in the order of 100°C/cm, a very good value that could be improved for the transition zone (1150° hot - 950° cold). There is also another good signal, the uniformity of temperature both on top surface and on edge one, sign that the rotating contact system has a good impact on the thermal uniformity.

### 6.2 Thermal efficiency

With the already introduced thermal losses is possible to calculate the efficiency of the direct heating process, in addiction to thermal losses due to the cooling of cold zone parts. Knowing that the theoretical surface losses were 92.7 kJ, is a good result that the measured energy losses of the non-homogenous simulation 99.8 kJ. These last losses are more than the theoretical one because they consider also thermal losses due to the process itself, the non-homogeneous heating, and not only the temperature prefixed of 900° that is the major component. So thermal efficiency is calculated by the difference of given energy at the work-piece and the thermal losses on the surface (radiation and convection) calculated each step and integrated on time:

![Flux diagram representing the thermal efficiency](image)

**Figure 4.2** Flux diagram representing the thermal efficiency
To calculate the efficiency only simulation quantities are taken into consideration. The whole energy given to the billet to heat up the work-piece is the simulated energy, made by direct and non-homogeneous energy. Like lost energy that is an integration in time of surface losses, simulated one is calculated by the integration in time of power inside the work-piece and then added in the two processes. We have already seen that energy for direct process is at least four time higher than the other, but is not the same for the energy losses. Energy losses are also divided in the two processes, not represented in previous flux diagram.

As said at beginning, the choice of develop only one hot zone instead of two has revealed the right choice, due to the high local heating the addiction of two zone instead of one does not interfere with the other. In terms of efficiency calculation we only need to sum another effect of hot zone heating for superimposition. Now is possible to go further with the calculation of thermal efficiency:

\[ E_{\text{losses}} = E_{\text{direct, lost}} + E_{\text{non-hom, lost}} = 82.2 + 99.8 = 182.1 \text{ kJ} \]
\[ E_{\text{simulated}} = E_{\text{direct}} + E_{\text{non-hom}} = 2064 + 45 \cdot 2 = 2154 \text{ kJ} = 0.598 \text{kWh} \]
\[ E_{\text{effective}} = E_{\text{simulated}} - E_{\text{losses}} = 2154 - 182.1 = 1972 \text{ kJ} \]
\[ \eta_{\text{thermal}} = \frac{E_{\text{effective}}}{E_{\text{simulated}}} = 1 - \frac{E_{\text{losses}}}{E_{\text{effective}} + E_{\text{losses}}} = 1 - \frac{182}{2154} = 0.915 \]

For the calculation of thermal efficiency were decide to have a time simulation as induction of above 100 seconds, so the first direct heating process last 58 sec, the second non-homogeneous conduction last the same time and values can be found in previous chapter. It is not available for most of companies a direct heating with that power requirements, so in a real longer heating the thermal losses for direct heating will be doubled.

The thermal losses of the second process are the higher because the billet is already at high temperature and irradiative losses are higher than convective one. The heating process require a theoretical energy of 1.79MJ, like the effective one calculated, so the result is verified.
6.3 Quantitative comparisons

In this paragraphs the differences are presented in terms of efficiency, energy and time requirements with respect to the previous studies.

Thermal efficiency:

$$\eta_{\text{thermal\_conduction}} = 0.915 \quad \eta_{\text{thermal\_induction}} = 0.82$$

The results are so different because of the extra-energy given to the work-piece in this study, considered as effective energy. Another point for the conduction is that the power is given uniformly in all the cross sections along the volume, so it is a uniform volume heating process, and then the non-homogeneous process makes the efficiency drop due to the time required with the hot work-piece. On the other hand, induction heating provides a lot of heating on the first skin depth when the steel is magnetic, rising up faster the temperature at start, but then it makes a lot of efforts to rise up further, increasing time for heating when the billet is already hot. To make the right comparison also conduction should be experimentally tested.

$$\text{Slope}_{\text{conduction}} = 100^\circ \frac{C}{cm} \quad \text{Slope}_{\text{conduction}} = 50^\circ \frac{C}{cm}$$

The slope of temperature in the transition zone is one of the most strength of non-homogeneous conduction heating process. Induction is good in both ways of efficiency and transition, direct conduction is good only in efficiency and non-homogeneous only in temperature slope transition, but together provide an hybrid heating with very good characteristics.

$$\text{Energy}_{\text{conduction\_theory}} = 0.496 \text{ kWh}$$

$$\text{Energy}_{\text{induction\_simulated}} = 0.485 \text{ kWh}$$

The energy required by theory to heat up the work-piece without losses, in both sides is similar and this is a good signal because means that the models are built with the same material properties, geometry, ect.
Here are compared the two energies required by the simulations, considering also the thermal losses. The result between them is similar. It proves that both the processes have a similar thermal efficiency, and the conduction one has to be reviewed the code in the exceeding part of thermal calculation efficiency formula. This is because is considered effective also the part of energy used to increase from 850° to 900°C, but it should be not be considered part of the effective energy.

\[
\begin{align*}
Energy_{\text{conduction}} &= 0.598 kWh \\
Energy_{\text{induction}} &= 0.59 kWh
\end{align*}
\]

\[
\begin{align*}
time_{\text{conduction}} &= 50/200 + 30/60 = 80/260 \text{ sec} \\
time_{\text{induction}} &= 80/100 \text{ sec}
\end{align*}
\]

The time is a really important parameter, where induction heating is much faster process since only one shoot can heat up all the billet. There is no chance to conduction heating to get closer because it is made by two processes, and the first is limited by current supply, the second by the low power transferred for the low path resistance of cross section.

### 6.4 Comments

The two processes of induction and conduction heating are similar when we are talking about electrical heating processes, but there are some strong differences when we look into details. In the following, the main strengths are listed for both processes, followed by their weaknesses:

**Strengths of conduction heating:**
- Flexible temperature design, due to the high local gradient of temperature
- Modular composition because it can be implemented after an already existing and working heating process
- Works with any type of metal
Strengths of induction heating:
- Low time process, due to the single shoot process
- Can transfer a lot of power when the steel is magnetic
- Already developed system and usual electrical heating treatment

Weaknesses of conduction heating:
- Maintenance for wear of sliding parts
- High current required for the low resistance
- Need another main process that pre-heat the work-piece

Weaknesses of induction heating:
- Fixed system, once time built cannot change
- Magnetic field confination
- Need special equipments to be supplied

6.5 Summary

This project was developed at Hannover in ETP, Institute of Electroheating of Leibnitz University. It has the purpose to carry out an investigation on the possibility to develop a non-homogeneous heating process using the conduction heating, and compare this study with previous work done on induction heating. It result the interest of an experimental work that demonstrate the strengths of this type of heating, previously summarized. The following work will regard:

I. Electrical efficiency implementing the contacts

II. Thermal investigation of cooling contact system that will deplete the theory of the “hot point” decreasing the efficiency

III. Optimization of:
   a. the temperature to start the process of non-homogeneous heating to get the lowest energy consumption
   b. process of rotating contacts for a wider hot zone in z direction, keeping high gradients

IV. Experimental work that confirms the simulations results
A1. Appendix

Here is reported the code post processor, text recalled at end of solve file, to understand how to save pictures, rename files in Ansys language.

```ansys
!*********************************************!
!              Postprocessor             
!*********************************************!
/post1

Temperatures in key points

csys,1

esel,s,mat,,200 !Temp superficial Top 1
   nsle
   nsel,r,ext
   nsel,r,loc,z,z1,z1+z_curr*3
   nsel,r,loc,x,x2,x2+eps
   nsel,r,loc,y,89.9,90
   *get,n_nodes,node,0,count !number of nodes
   *dim,wp_y,,n_nodes,1
   *dim,wp_temp,,n_nodes,1
   *get,nmin1,node,0,num,min
   *do,ii,1,n_nodes
     *get,Ycoo,node,nmin1,loc,y
     *set,Ycoord,Ycoo
     *vfill,wp_y(ii),data,Ycoord
     *get,temperat,node,nmin1,temp
     *vfill,wp_temp(ii),data,temperat
     *get,nmin1,node,nmin1,nxth !next higher node number
   *enddo
   *vscfun,Temp1,max,wp_temp !max node temperature=Temp1

  csys,1
  esel,s,mat,,200 !Temp middle 2
     nsle
     nsel,r,ext
     nsel,r,loc,z,z1,z1+eps
     nsel,r,loc,x,x1,x1+eps
     *get,nout,node,0,num,min
     *get,Temp2,node,nout,temp

  csys,1
  esel,s,mat,,200 !Temp superficial Top 3
     nsle
     nsel,r,ext
     nsel,r,loc,z,z2,z2+eps
     nsel,r,loc,x,x2,x2+eps
     nsel,r,loc,y,89.9,90
     *get,nout,node,0,num,min
     *get,Temp3,node,nout,temp

  csys,1 !Temp superficial dx 4
```
esel,s,mat,,200
nsel
nsel,r,ext
nsel,r,loc,z,z1,z1+eps
nsel,r,loc,x,x2,x2+eps
nsel,r,loc,y,0,1
*get,nout,node,0,num,min
*get,Temp4,node,nout,temp

!--------------------------------------------------------
! temperatures images

esel,s,mat,,200,270
nsel,all
PLNSOL,TEMP,!
   esel,s,mat,,200,270
   nsel,all
   /PLOPTS,DATE,0
   /VIEWS,1,1,-1
   /ANG,1
   /auto,1
   /rep,FAST
   /show,WIN32C
   /contour,1,15,auto,10
   /show,JPEG,0
   JPEG,QUAL,100,
   JPEG,ORIENT,HORIZ
   JPEG,COLOR,2
   JPEG,TMOD,1
   /gfile,2400,
   /cmap,_TEPCMAP_,CMP,,save !color map
   /rgb,INDEX,100,100,100,0
   /rgb,INDEX,0,0,0,15
   plnsol,temp,,0,
   /cmap,_TEPCMAP_,CMP !color mapping table
   /delete,_TEPCMAP_,CMP
   /show,CLOSE
   /device,vector,0
!/rename,%simulation%000,.jpg,,sim_front_%freq%_%i_re%_%zeit%,jpg,
   result,%y_angle%_%d2dnext(1,2)%_%freq%Hz%_%i_re%A
   /rename,%simulation%000,.jpg,,sim_front_%freq%_%i_re%_%zeit%,jpg,result
   /VIEW,1,1,-1
   /ANG,1
   /REP,FAST
   /ZOOM,1,RECT,-0.942048,0.156248,-0.50687431445,-0.147290907485
   /contour,1,15,auto,10
   /show,JPEG,0
   JPEG,QUAL,100,
   JPEG,ORIENT,HORIZ
   JPEG,COLOR,2
   JPEG,TMOD,1
   /gfile,2400,
   /cmap,_TEPCMAP_,CMP,,save !color map
   /rgb,INDEX,100,100,100,0
   /rgb,INDEX,0,0,0,15
   plnsol,temp,,0,
   /cmap,_TEPCMAP_,CMP !color mapping table
   /delete,_TEPCMAP_,CMP
   /show,CLOSE
   /device,vector,0
!/rename,%simulation%000,.jpg,,sim_edge_%freq%_%i_re%_%zeit%,jpg,
   result,%y_angle%_%d2dnext(1,2)%_%freq%Hz%_%i_re%A
   /rename,%simulation%000,.jpg,,sim_edge_%freq%_%i_re%_%zeit%,jpg,result
!--------------------------------------------------------
!indicators of uniformity of temperature:
grad1=(Temp1-Temp2)/(y2-y1)  !top/mid y
grad2=(Temp1-Temp3)/(z2-z1)  !surface z
grad3=(Temp4-Temp2)/(x2-x1)  !ext/mid x

*CREATE,ansuitmp
*CFOPEN,'hhh_%freq%_%i_re%','txt',,append
*VWRITE,hhh,zeit,Temp1,Temp2,Temp3,Temp4,grad1,grad2,grad3
        (2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F10.3,2x,F10.3)
*CFCLOSE
*END
/INPUT,ansuitmp

!add 1 to constant cycle and save it
hhh=hhh+1
save,solu_therm_db,db,,all

*CREATE,ansuitmp
*CFOPEN,'h_cycle','txt',
*VWRITE,kkk,jjj,hhh,zeit,move,power,energy,losses,en_lost
        (2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0,2x,F8.0)
*CFCLOSE
*END
/INPUT,ansuitmp

finish
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