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“The Financial Portfolio Construction and the VIX
Index: From a Theoretical Outlook to a Practical
Application.”

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**Introduction**

In recent times, the volatility topic is returned to the center of financial market worries. The volatility explains the risk related to the price fluctuations over time. The volatility calculated on options, the implied volatility, explains the expectations on the exercise of options; if it is high, means that the market and its participants expect, with high probability, big fluctuations in the returns of the options’ underlying. The contrary, if it is low. The most important index based on the implied volatility is the VIX index, introduce exactly 25 years ago. Already before the financial crisis of 2008, numerous studies have been conducted on this index; market operators start to know the VIX index and from 2004 started to use instruments constructed on it. Then, increasingly, these instruments have been produced, arriving to create funds easily available also for retail investors.

Overtime, many studies analyzed the possibility of adding the VIX index in a financial portfolio, primarily to improve diversification, returns and standard deviations.

Combining these aspects, the object of this work is to understand which type of strategies is the most efficient to exploit the properties of the VIX instrument, represented by an index and an ETF, in a financial portfolio completely invested in mutual funds.

The path of the study is composed by 3 sub analysis, aggregated in a hierarchical scheme.

The first strategy is the following: “buy” each mutual funds of the portfolio with weights deriving from a past analysis and “hold” the portfolio until the end of the period considered, form January 2007 to June 2018; this can be compared with the same analysis made on a portfolio with a static weight given to the VIX instrument, set ex-ante with respect to the starting life of the portfolio.

The second strategy passes to a shorter-term period analysis that consider the maximum period of time to hold the portfolio equal to 1 year; from this strategy two views follow: the first is, in an ex-ante perspective, deciding to set the weight of the VIX instrument to a static amount; in the second is, in an ex-post perspective, understanding, setting the best allocation of the portfolio and the most efficient weight of the VIX instrument, in which measures the abilities of the VIX index can improve the portfolio.

The third strategy concludes to an even shorter-term period, approaching towards a trading view, and in addition to past studies and history, is added the technical analysis framework, implemented through the use of moving averages.
The work title reveals the thesis structure:

- Part 1 describes the theoretical context, referring to the Markowitz’ theory, in which an investor stands when decides to invest the money in the financial market.
- Part 2 focuses on a practical case that involves an investor, with specified characteristics, that employs part of the wealth in a portfolio composed exclusively by mutual funds representing the most common asset classes, in the period January 2007 to June 2018. In this Part the first two of the three strategies are presented; the buy and hold and the tactical strategy at 1 year. The analysis occurs considering how performance, standard deviation, correlations and other comparison tools change over time and with respect to the portfolio without the addition of the volatility instrument.
- Part 3 is dedicated to the third strategy, the tactical allocation of the VIX index, that tries to catch signals that allow the investor to understand ex-ante when should buy or sell the volatility instrument of the financial portfolio, through the implementation of the technical analysis.
Part 1 – The theoretical framework: the portfolio construction, mutual funds and ETFs and the VIX index.

The Part 1 includes the theoretical framework of the work. It looks through the analysis of the investor’s portfolio, from the choice of asset classes to the monitoring of the entire portfolio. Then, this part focuses on the typical instruments that the major part of the investors can easily utilize in their portfolio: mutual funds and ETFs. Finally, there is an overview of the most common volatility index, the VIX.

The Part 1 is composed by three chapters:

- Chapter 1. The portfolio construction theory.
- Chapter 2. Mutual funds and ETFs components.
- Chapter 3. The CBOE volatility index: the VIX.

Chapter 1. THE PORTFOLIO CONSTRUCTION THEORY.

The investment process, needed to create a financial portfolio, reaches its fuller meaning in the first step: to collect of investment assets. The investor, in this step, has to make two types of decisions: the asset allocation decision, the choice between different asset classes, keeping in mind the possibility to place money in safe assets, as bank accounts or money market securities, or in risky assets, as stocks and bonds; and the security selection decision, the choice of which specific securities, within each asset class, are included in the portfolio.

Once a portfolio is constructed, it can be rebalanced, in more or less long periods of time, by changing the weights in the different asset classes that composed it; for example, by selling existing securities and using the amount of money received from the sale for buying new securities or by investing in existing or new securities.

1.1 The financial market components: asset classes and securities types.

For going through the asset allocation and the security selection decisions, I need to deepen the financial market components: asset classes and financial securities.

An asset class is a container of securities that show similar characteristics, behaves similarly in the market and is subjected to the same laws and regulations.

The primarily asset classes are four:

- the Money/Cash: is part of the fixed-income market. It includes very short-term debt securities that usually are highly marketable, as treasury bills, government tool to raise money, certificates of deposit, deposited next to a bank, commercial paper, issued by a large well-known company.
• the Bond: includes longer term borrowing or debt instruments with respect to the Money asset class, as treasury notes with maturity up to 10 years and bonds with maturity between 10 to 30 years, inflation-protected treasury bonds, linked to an index cost of living, corporate bonds, issued by private firms.

• the Equity: includes common stocks as ownership shares, that entitled to be owner of the company in a measure equal to the weight of instrument hold, and preferred stock, similar to both equity and bonds.

• the Derivative: includes instruments which the payoff depends on the value of other assets, as options, that give its holder the right to buy (call option) or sell (put option) an asset for a specified price on or before a specified expiration date; futures, that allow to deliver an asset or its cash value at a specified delivery or maturity date for an agreed-upon price to be paid at contract maturity.

Vehicles more useful for retail investors are mutual funds and Exchange-Traded Funds. Also these instruments are included into the asset classes described above. The Chapter 2 of the Part 1, is entirely dedicated to these two financial instruments.

1.2 The financial portfolio construction: the point of view of Markowitz.
Going through the construction of a financial portfolio, I consider the point of view of the most advanced economist, for times lived and innovations proposed, in terms of a financial portfolio construction, and the Nobel Memorial Prize in Economic Science of 1990: Harry Markowitz. With his paper “Portfolio Selection” published in *The Journal of Finance* in 1952, Markowitz gave a great contribution in Modern Portfolio Theory, studying the effects of asset risk, return, correlation and diversification on probable investment portfolio returns.

In the “Portfolio Selection” paper, Markowitz focuses the attention on two stages: the observation and experience arriving to some beliefs about the future performances of some securities and the study of relevant beliefs about future performances arriving to the choice of the portfolio.

The fundamental principles of the Markowitz portfolio theory are:

• the investor prefers a security or a portfolio which has maximum return and minimum risk, for maximize their wealth;

• the investment period is unique;

• transaction costs and taxes are not considered, activities are perfectly divisible;

• the expected return and the standard deviation are the only two measures to consider;
• the market is perfectly competitive, free and quickly.

According to Markowitz, the investor should diversify his portfolio among securities that give him the maximum expected return and the minimum variance. Or, at least, between two securities having the same expected return, the investor should choose the security with lower variance; instead, between two securities having the same variance, he should choose the security with higher expected return.

Therefore, Markowitz considers the Expected returns – Variance of returns (E-V) rule, that is the evaluation of the trade-off between return and risk on investments. The rule includes two measures: in one hand, there is the Expected return, or simply the mean of a portfolio; on the other hand, the Variance of returns, or better the standard deviation of a portfolio.

The mean
The mean is the arithmetic average of rates of returns; the mean of the portfolio is the weighted average of rate of return of each securities that are included in the portfolio.

\[
E(r) = \frac{1}{n} \sum_{i=1}^{n} w_i \times r_i
\]

where \(E(r)\) is the Expected returns, \(w_i\) is the weight of each security, \(r_i\) the rate of return of each security, \(n\) the number of securities.

The variance and the standard deviation
The standard deviation is a measure of risk or volatility because is the square root of the expected value of the squared deviations from the mean, that is the variance:

\[
\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} [r_i \times E(r)]^2
\]

It does not represent only the downside risk, but can represents risk linked to good or bad deviations from the mean: the volatility.

The graphical representation of the E-V rule is called Efficient Frontier, that is the boundary of the set of feasible portfolios that have the highest expected return for a given level of risk or the lowest risk for a given level of expected return. “Portfolios that lie below the Efficient Frontier are sub-optimal because they do not provide enough return for the level of risk. Portfolios that cluster to the right of the Efficient Frontier are also sub-optimal because they have a higher level of risk for the defined rate of return”.

---

The benchmark

Usually, an investor’s portfolio is compared to a model portfolio, for observing if it performs better or worse with respect to the model one: the benchmark portfolio.

In general, the benchmark is “a standard by which something is measured. For example, bond yields are generally compared to benchmark yields on U.S. Treasury securities of similar maturity. Mutual fund performance is often compared to changes in the Standard & Poor’s 500 Stock Index”\(^2\).

The risk-free asset

Another comparison tool, usual in many formulae, is the risk-free asset.

The risk-free asset is an asset that is free from risk; the risk-free rate is a theoretical rate of return of an investment with zero risk. Usually, the more used risk-free asset is Treasury bill, because is an instrument issued by a government. But, “in practice, most investors use a broad range of money market instruments as a risk-free asset. All the money market instruments are virtually free of interest rate risk because of their short maturities and are fairly safe in terms of default or credit risk”\(^3\).

---


The diversification

Another important element in the Modern Portfolio Theory is the concept of diversification. Diversification does not mean only investing in a large number of securities. It considers the correlation coefficient, that is a statistical measure that explains, in numerical terms, if two variables move in the same direction or in the opposite one.

The covariance

The covariance between two different securities is:

\[
Cov(x, \eta) = \frac{1}{n} \sum_{i=1}^{n} (r_{x_i} - X)(r_{\eta_i} - Y)
\]

where \(r_{x_i}\) is the return of one security, \(x\) and \(X\) is its mean, \(r_{\eta_i}\) is the return of another security, \(\eta\) snd \(Y\) is its mean.

The correlation coefficient

The correlation coefficient is:

\[
P_{x\eta} = \frac{Cov(x, \eta)}{\sigma_x \sigma_\eta}
\]

where Cov\((x, \eta)\) is the formula above, \(\sigma_x\) is the standard deviation of security \(x\), \(\sigma_\eta\) is the standard deviation of security \(\eta\).

If the correlation coefficient is +1, the diversification does not work, if it is 0, the diversification works, if it is -1, the diversification is “optimal”.

The types of risk and diversification

Related to the diversification issue, are the different types of risk existing in a financial portfolio. There exist two types of risk:

- the firm-specific risk or unique risk that is diversifiable, with diversification, since it considers, for example, the small amount of securities invested in a particular company or industry;
- the systematic risk or market risk that is not diversifiable, even with diversification, since it is referred to the volatility, the daily fluctuations of prices.
For what concerns the types of diversification, they can be divided in:

- simple, with the increase of securities in the portfolio, given the return, the risk decreases. Initially, the benefit is more sensitive, then the diversification loses benefit with the increase of number of securities in the portfolio; a practical example is the investment in mutual funds.
- efficient, is more effective when the correlation coefficient between securities is low.
- time, the standard deviation decreases with the duration of the investment period.
- international, usually foreign activities are not completely related with those of domestic returns.

Going forward to the practical portfolio construction, in particular in the choice of securities, the investor should consider two conditions:

- market conditions or external conditions, consider return, risk and correlation between asset classes and securities, already covered in this work;
- social-economic conditions of the investor, primarily, consider its risk attitude, usually tested with the MiFID questionnaire, through which the investor can understand which level of risk can tolerate.

### 1.3 Portfolio management policies.

In the investment process, in choosing the securities to be included in the portfolio, and after, when the portfolio has to be monitored, can be used different management policies, related to the conditions described above. There exist three portfolio management policies: strategic, tactical and dynamic.

#### Strategic policy

The strategic policy accepts the hypothesis of efficient market, points on the diversification and ignores the timing. There are two types of strategic policy:

- the passive one, or buy and hold, that replicates the reference portfolio and does not require any trading activities;
the constant mix, that focuses on specified characteristic combinations in percentage term of the invested value in each class (for example, 60% bonds and 40% stocks); it requires trading activity to save the ratio between stocks and bonds (Perold, Sharpe, 1988).

This policy is neither in function of market changes nor in function of social-economic conditions changes of the investor.

Tactical policy
The tactical policy has the objective to catch occasions that the market offers in terms of timing and the selectivity of securities or classes that over or under-perform with respect to the market.
This policy is in function only of market changes.

Dynamic policy
The dynamic policy proposed to make better with respect to the market and, at the same time, disposes of a desired minimum capital at the maturity to protect the capital itself from unwanted losses.
This policy is in function only of social-economic conditions changes of the investor.
Chapter 2. MUTUAL FUNDS AND ETFS COMPONENTS.

In the practical application of my work, in Part 2, I made use of these two elements: mutual funds, mainly, and ETFs. This chapter contains description, structure, asset allocation, monitoring and analyzing methods related to mutual funds and ETFs.

2.1 Mutual Funds.

The mutual fund is an investment vehicle that is evolved during the years, from closed-end companies started in the Netherlands in 1822 by King William I, to the modern type created in Boston in 1924 by the Massachusetts Investors’ Trust. Today, a mutual fund is typically an open-end fund, held by an open-end investment company, which stands ready to buy back its shares from investors at the end of every business day at the Net Asset Value (NAV). The NAV is calculated by subtracting the fund’s liabilities from the current market value of its assets and dividing by the number of shares outstanding. Typically, investors can buy shares of mutual funds daily at the NAV. From the collection of this money, the manager of the fund invests, on behalf of investors, in stocks, bonds or other assets. With this mechanism, each investor owns shares of the fund.

The objective of mutual funds can be various: international, regional, country-specific or specific type of company. They take different risk levels: from low risk (mutual funds investing mostly in cash) to high risk (mutual funds investing in new companies or uncertain markets). Mutual funds can be actively managed, so the fund manager is paid to study the market and to buy assets to arrive at the best objective; other can simply mimic an index present on the market using a mix of investments.

There are advantages and disadvantages of mutual funds with respect to directly investing in individual assets or securities. Advantages of investing in mutual funds are:

- they are the simplest and least expensive way to have access to different markets and securities and to have participation in investments usually available only to larger investors;
- the fact that there is a unique management that handles different issues, from the buying and selling of the assets to collecting income, becomes a service and a convenience for investors;
- since money of different investors are pooled together, they also share costs and benefits from economies of scale, which allow for lower trading costs;
• can be less risky with respect to investing money in individual companies and so having an increased diversification;
• investors rely on professional money management that take decisions on the future activities of the fund;
• transparency for large disclosure requirements, daily valuation, liquidity, regulatory supervision and accountability.

There are also disadvantages of investing in mutual funds:

• there are fees and expenses that decrease returns for investors either reducing the value of investors account or reducing the NAV because are paid by the fund;
• loss of control over timing in recognizing gains, less predictable income and no opportunity to customize.

2.1.1 The creation and controlling mutual funds portfolio.
For selecting which mutual funds the investor can include in the portfolio, the investor should consider various issues. Pozen and Hamacher (2011) propose these elements proper of the investor:

• liquidity needs, as flexibility ability of the investor in redemption of the investments;
• time horizon, that can result in short, medium or long term and defined in months or years;
• return expectations, thought as capital appreciation, earnings generation, tax saving or a combination;
• risk tolerance, linked, first to the investor as a subject, then to the investment aim, for example, high risk tolerance if the objective is the capital appreciation and to the time horizon, for example, high risk tolerance for long-term investment horizon

2.1.2 The analysis of mutual funds’ performance.
For analyzing mutual funds’ performance, the investor can use a series of quantitative performance metrics.
**The Coefficient of Variation (CV)**

The Coefficient of Variation (CV) measures the ratio between the portfolio’s standard deviation ($\theta_p$) and the portfolio’s return ($r_p$):

$$CV_p = \frac{\theta_p}{r_p}$$

The CV gives a numerical value of the risk or volatility for the investor per unit of return; here, the standard deviation represents the total risk of the mutual fund portfolio, including both the market risk and the firm-specific risk. It’s a dimensionless measure. Portfolio with lower CV is better than another with higher CV.

**The Sharpe ratio (S)**

The Sharpe ratio (S), a formula introduced by William Sharpe in 1966, measures the ratio between the difference between the portfolio’s return ($r_p$) and the risk-free rate ($r_f$) and the portfolio’s standard deviation ($\theta_p$):

$$S_p = \frac{r_p - r_f}{\theta_p}$$

The Sharpe ratio gives the return net of the risk-free rate in relation to the risk or volatility of the portfolio. Also, this ratio is a dimensionless measure. Portfolio with higher Sharpe ratio is better than another with lower ratio.

**The Treynor ratio (T)**

The Treynor ratio (T), a formula introduced by Jack L. Treynor in 1965, which contains a new measure: beta, or the systematic risk of the portfolio ($\beta_p$). “Beta is a measure of sensitivity of an investment or portfolio to movements in the overall market and is a linear risk measure”4.

The Treynor ratio measures the ratio between the difference between the portfolio’s return ($r_p$) and the risk-free rate ($r_f$) and the beta ($\beta_p$):

$$T_p = \frac{r_p - r_f}{\beta_p}$$

The Treynor ratio gives the return net of the risk-free rate in relation to only the systematic risk, the non-diversifiable risk. Also this ratio is a dimensionless measure. Portfolio with higher Treynor ratio is better than another with lower ratio.

---

The Modigliani risk-adjusted performance ($M^2$)

Since the measures described above are all difficult to interpret because are dimensionless, Franco Modigliani and Leah Modigliani, his granddaughter, in 1997, try to overrun these drawbacks developing the Modigliani risk-adjusted performance or $M^2$.

The $M^2$ measures the Sharpe ratio ($Sp$) times the market’s standard deviation ($\theta m$) plus the risk-free rate ($rf$):

$$M^2 = Sp \times \theta m + rf$$

It is closely related to the Sharpe ratio, but it is calculated in percentage terms.

The Information ratio (IR)

Another tool for measuring the performance of mutual funds, is the Information ratio (IR). It measures the ratio between the difference between the return of the portfolio ($rp$) and the return of the benchmark ($rb$), called Tracking error, and the difference between the standard deviation of the return ($\theta p$) and the standard deviation of the benchmark ($\theta b$):

$$IR = \frac{rp - rb}{\theta p - \theta b}$$

It shows the over or under-performance of the mutual fund with respect to the performance of the benchmark.

Portfolio with higher Information ratio is better than another with low ratio.

2.1.3 Asset pricing models of financial markets.

To understand the relationship between the return of the portfolio, including mutual funds, and its riskiness, over time, various models have been developed. The most known and used are the following two: the Capital Asset Pricing Model (CAPM) and the Fama & French model.

The Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) is a regression method developed independently by, first of all, Harry Markowitz (1952, 1959), then Jack L. Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965a, b) and Jan Mossin (1966). It appears in the seminal paper of Michael Jensen (1968) and measures the mutual fund performance relying on three inputs. The risk-free rate ($Rf$), market risk premium ($Rm$) and beta ($\beta j$):

$$E(Rj) - Rf = \alpha j + \beta j [E(Rm) - Rf]$$

The $\alpha j$ is called the Jensen’s Alpha, introduced by Michael Jensen in 1968. The intercept measures, in percentage terms, the over or under-performance that an actively managed mutual fund reached, relative to the risk.
The Fama & French model

In 1977, Richard Roll analyzed the CAPM and made one important critique: the market portfolio is unobservable, because it should include every single asset existing in the financial market. It is not possible calculate returns of all investments available in the market. Starting from this the Roll’s critique, in the academic world, alternative multifactor models are considered, for better explain the financial asset performance. Eugene Fama and Kenneth French, in 1993, developed and identified three common risk factors related to equity market:

- beta ($\beta$), analogous to the CAPM beta;
- Small Minus Big (SMB), measures the historical excess returns of small capitalization over big capitalization;
- High Minus Low (HML), measures the historical excess returns of value stocks (high book-to-market ratio) over growth stocks (low book-to-market ratio).

These factors are obtained from the combination between portfolio of ranked stocks and historical market data. The coefficients of SMB and HML are obtained by linear regressions and can be positive or negative.

Stocks with small market capitalization perform better than stocks with big market capitalization (size effect); stocks with low book-to-market ratio perform better than stocks with high book-to-market ratio (value effect).

In 2015, the two academics extended the model, including two other factors related to bond market:

- Robust Minus Weak (RMW), measures the returns of firms with robust operating profitability over those with weak operating profitability;
- Conservatively Minus Aggressively (CMA), measures the returns of firms that invest conservatively over those that invest aggressively.

Stocks with robust operating profitability perform better than stocks with weak operating profitability (profitability effect); stocks investing aggressively perform better than stocks investing conservatively (investment effect).

The five-factor model is:

$$ E(R_{jt}) - R_{ft} = \alpha_j + \beta_j [E(R_{mt}) - R_{ft}] + \sigma_j SMB_t + \gamma_j HML_t + \eta_j RMW_t + \zeta_j CMA_t $$

If the exposures to the five factors capture all variation in expected returns, the intercept $\alpha$ is zero for the portfolio.
2.1.4 Downside measures of risk.

From the real data of the financial market, it can be observed that returns of securities are not normally distributed and not symmetrical; this means that it needs to distinguish between the upside risk, that is the risk relative to the extent of gains, and the downside risk, that is the risk relative to the extent of losses.

For observing the last type of risk, there are three indicators: the Value at Risk (VaR), the Expected Shortfall (ES) and the Maximum Drawdown (MDD).

**The Value at Risk (VaR)**

The Value at Risk (VaR) is indicative of the probability of the maximum potential loss of a security or an entire portfolio, considering a specified time horizon, that can be one day as one year, and a certain level of confidence, that typically is 95% or 99%. The level of confidence is the percentage of the protection from the risk of loss; if the level of confidence is very high, this means that it is high also the capacity to bear losses of the market.

The VaR can be calculated with different methods: Variance-Covariance approach, Asset/Beta Normal, Local Valuation, Simulation approach, Monte Carlo simulation and Historical simulation.

**The Expected Shortfall (ES)**

Usually, the VaR does not give the correct result; it can be that two portfolios, with different probability distribution for gain and loss and different risks, result in the same VaR. For this reason, it has been introduced another measure: the Expected Shortfall (ES), also called Conditional VaR (CVaR). The CVaR measures the amount of tail risk an investment portfolio can have.

“While VaR represents a worst-case loss associated with a probability and a time horizon, CVaR is the expected loss if that worst case threshold is ever crossed.”

**The Maximum Drawdown (MDD)**

The Maximum Drawdown is a risk indicator that represents the maximum accrued loss, in the period of analysis, with respect the previous peak. Related to this concept there is the period for recovering the loss that is the number of days/months/years needed to bring back the portfolio or instrument considered from its minimum to its initial peak.

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2.2 Exchange-Traded Funds (ETFs).

The first real Exchange-Traded Fund was introduced in 1993 by the State Street Global Investors, called S&P 500 Trust ETF.

An ETF is typically structured as an open-fund and, as the name suggests, its shares are traded on stock exchanges intraday at prefixed market prices. Investors can buy these shares through a broker, a licensed professional person.

The major part of ETFs is linked to financial indices, such as domestic, sector, regional or international indexes and tries to replicate their performance.

There are advantages and disadvantages of investing in ETFs.

Advantages are:

- investors can have access to a wide market;
- investors have low administrative costs compared to other investment products because ETFs follow a passive management investment strategy;
- ETFs are cheaper than other products on the same indexes;
- ETFs are flexible because investors can buy and sell shares at every time in the trading day at a market price;
- investors can use ETFs for hedging purposes;
- ETFs are tax-efficient because of low turnover for fewer taxable gains;
- ETFs offer transparency because they are priced frequently in trading day and published their holdings daily.

There are also disadvantages of investing in ETFs:

- ETFs are not permitted in retirement and pension funds;
- ETFs are only passive managed funds so only mimic the market and do not try to over-perform it.
### 2.2.1 ETFs replication mechanisms.

There exist many methods to replicate the index, represented in Figure 2.1.

**Replication methods of ETFs in comparison**

<table>
<thead>
<tr>
<th>Replication method</th>
<th>Physical</th>
<th>Synthetical</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
<td>The index is replicated 1:1</td>
<td>The index replicates the index by using a financial derivative (swap)</td>
</tr>
<tr>
<td><strong>Underlyings</strong></td>
<td>Equities, Bonds</td>
<td>Equities, Bonds, Commodities, Money Market (EONIA etc.), Short and Leverage indices</td>
</tr>
<tr>
<td><strong>Typical characteristics of index components</strong></td>
<td>Liquid securities</td>
<td>Illiquid securities, Liquid and illiquid securities, investment restrictions (trade restrictions, taxation), different time zones</td>
</tr>
<tr>
<td><strong>Typical number of securities in the index</strong></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td><strong>Sample indices</strong></td>
<td>FTSE 100, Eurostoxx 50, Dow Jones 30</td>
<td>MSCI World, MSCI Emerging Markets</td>
</tr>
</tbody>
</table>

Figure 2.1: Replication methods of ETFs in comparison. Available on: <https://www.justetf.com/uk/academy/etf-replication-methods.html> [Date of access: 28/11/2018].
Chapter 3. THE CBOE VOLATILITY INDEX: THE VIX.

The Chicago Board Options Exchange (CBOE), the world’s largest options market, in 1993 launched the publication of the real-time VIX index. The VIX index is an index measuring the 30-day expected volatility of the United States stock market, calculated on implied volatility of options contract prices. It is called, also, the “fear index” or the “fear gauge”, because when investors are afraid and want to protect themselves from market drops, start to buy put options, representing an insurance tool for their portfolio; the more is the demand in put options, the higher becomes the price and consequently the higher becomes the VIX index.

3.1 The volatility.
Before entering in the specific of the VIX index, a brief overview on the volatility topic follows. The volatility is the measure of how much the price of an asset goes up and down, in positive and negative direction. It is calculated with the standard deviation formula seen in Chapter 1 Section 2. There is high volatility when prices change quickly; there is low volatility when prices do not change very much. There are different types of volatility:

- intraday volatility, that measures the price moves in the market during a trading day.
- historical volatility, that measures daily changes in prices and represents the standard deviation calculated on historical prices;
- implied volatility, calculated in the Black-Scholes formula\(^6\) with the \(\theta\) symbol; it measures the estimated volatility of an asset’s price, that derives from the pricing options.

The VIX index refers to the implied volatility type.

3.2 The VIX History.
The first Section entirely dedicated to the VIX index is dedicated to its history path. It can be summarized as following:

- in 1987, in the paper of Brenner and Galai published later (1989), was introduced for the first time the “Sigma Index”, writing “Our volatility index, to be named Sigma

\[ C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)} \]

where \(d_1 = \frac{\ln(S_t/K) + (r + \theta^2/2)(T-t)}{\theta \times \sqrt{T-t}}\) and \(d_2 = d_1 - \theta \times \sqrt{T-t}\)
Index, would be updated frequently and used as the underlying asset for futures and options... A volatility index would play the same role as the market index play for options and futures on the index.”\(^7\). After that, they developed their idea in further researches.

- in 1992, the American Stock Exchange started to conduct studies on a volatility index, considering the “Sigma Index”. “SI would be an underlying asset for futures and options that investors would use to hedge against the risk of volatility changes in the stock market.”\(^8\)
- in 1993, on January 19, the CBOE announced the launch of the VIX, measured on S&P 100 Index (OEX) option prices. The VIX was developed by Robert E. Whaley, a professor expert in derivative contract valuation and risk management and market operation.
- in 2003, the VIX structure changed; thanks to the working with Goldman Sachs, the CBOE developed further calculations and changed the underlying from the S&P 100 to a broader index, the S&P 500 index (SPX).
- in 2004, on March 26, there was the first trade in futures on the VIX index in the CBOE Futures Exchange (CFE).
- in 2006, on February, there was the launch of options on the VIX.

### 3.3 The VIX development.

As explained in the history section of this chapter, the VIX is initially developed by Robert E. Whaley in 1993 on behalf of the CBOE. The first version of the VIX was based on S&P 100 (OEX) eight option contracts. That is because at the time was the most traded index, so those options reflected the stock market in the United States.

In 2003, this methodology changed and the VIX was linked to the S&P 500 index (SPX) and an increased number of options.

#### 3.3.1 The S&P 500 index (SPX).

The S&P 500 index was realized by Standard and Poor’s corporation, in 1957. It tracks the trend of 500 United States companies with the higher market capitalization.

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In 2017, the industry composition of the S&P 500 index was:

![S&P 500 Index - 2017 composition](image)

Figure 3.1: The composition of S&P 500 Index at 31/12/2017. Available on: <http://siblisresearch.com/data/sp-500-sector-weightings/> [Date of access: 28/11/2018].

In 2017, the 10 largest companies in the S&P 500 (with a weighted market capitalization) were Apple, Microsoft, Amazon, Berkshire Hathaway B, Facebook, JP Morgan Chase, Johnson & Johnson, Exxon Mobil, Alphabet C (formerly Google), and Alphabet A.

### 3.4 The VIX calculation.

The generalized formula of the VIX calculation, available on the white paper in the website of the CBOE, is:

\[
\theta^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]^2
\]

where:

- \( \theta = \frac{VIX}{100} \),
- \( T \) = time to expiration, calculated precisely (open of trading at 08:30 am Chicago time for monthly options and close at 03:00 pm for weekly options);
- \( F \) = Forward index level derived from index option prices;
- \( K_0 \) = First strike below the forward index level, \( F \);
- \( K_i \) = Strike price of \( i^{th} \) out-of-the-money option; a call if \( K_i > K_0 \) and a put if \( K_i < K_0 \); both put and call if \( K_i = K_0 \);
- \( \Delta K_i \) = average strike price increment – half the difference between the strike on either side of \( K_i \): \( \Delta K_i = \frac{K_{i+1} - K_{i-1}}{2} \);
- \( R \) = Risk-free interest rate to expiration, represented by the bond-equivalent yield of US T-bills;
- \( Q(K_i) \) = the midpoint of the bid-ask spread for each option with strike \( K_i \).

Going inside the formula, “the components of the VIX Index are near- and next-term put and call options with more than 23 days and less than 37 days to expiration. These include SPX options with “standard” 3rd Friday expiration dates and “weekly” SPX options that expire every Friday, except the 3rd Friday of each month. Once each week, the SPX options used to calculate the VIX Index “roll” to new contract maturities.”

The options used have non-zero bid to guarantee liquid options and are selected at the money strike up for call options and down for put options, until two consecutive zero bid strikes are reached. Therefore, the set and the number of options included in the VIX formula vary over time in line with changes in the S&P 500 index.

The 30-day variance, \( \theta^2 \), is measured interpolating the variances of the two expirations, weighted considering how close or far each expiration is from the 30-day goal. The weights must sum 1.

The last step, for having the VIX index, is to take the squared root of the variance, that is arrive to the standard deviation measure, and multiply by 100.

---

3.5 The VIX index and the S&P 500 index.

The most important peculiarity related to this volatility index is the issue that the VIX index has an inverse relationship with respect to the S&P 500 index, that is, they move in opposite directions. This means that the correlation coefficient is negative, lower than 1. Precisely, the correlation between the two indices in the period June 2004 – June 2018 is -0.674.

Figure 3.2: S&P 500 index and VIX index returns. Historical data available on: <https://it.investing.com> [Date of access: 28/11/2018].

3.5.1 The put-call parity.

The explanation of this relationship can be found in the so called “put-call parity”, that is in the nature of purchasing options. When the market is in a trouble situation, a net buying of put option wave happens, which increases the implied volatility of both put and call contracts. The put-call parity can be explained as follows: prices of put and call options that have the same strike price and expiration dates are related because there exists the ability to create synthetic positions in one option through combining to other option with the underlying stock. If prices differ, there is an arbitrage opportunity. Since, it is possible to replicate the payout of a long call combining a put and a stock position, this means that these two structures must to be equivalent. If they are not equal, one may buy the cheapest and sell the most expensive one. This is an arbitrage trade since an instant profit is realized through prices difference.
This simplistic explanation leads to understand in which way the put-call parity affects the VIX index. The last is linked to the implied volatility that, in turn, is linked to the purchasing of call and put options. The more is the demand of call options, the more is the increase in the put options prices and vice versa. Therefore, higher demand for both contracts involves higher implied volatility for both contracts.

Another issue related to the inverse relationship between the VIX index and the S&P 500 index that derived from the put-call parity, is in which context the market is in a particular moment: bullish versus bearish markets.

When the market is bull, this means that prices are going up and so investors do not purchase call options.

When the market is bear, this means that prices are going down and the market is in trouble, so investors demand for put options for protecting them in the turbulent situation. At this moment, considering the put-call parity, the implied volatility for both S&P 500 call and put options starts to increase and consequently the VIX index increases.

### 3.5.2 The regression.

To understand even better the relationship between the VIX index and the S&P 500 index, considering the regression calculated in the paper of Robert E. Whaley (2008), I tested the following regression:

\[
VIX_{t_t} = \alpha_0 + \beta_0 S^P_X_t + \varepsilon_t
\]

where \( VIX_{t_t} \) represents the change in VIX index prices, \( \alpha_0 \) is the intercept term of the regression, \( \beta_0 \) is the coefficient term that gives the an information about how the S&P 500 index explains the change in VIX prices, \( S^P_X_t \) represents the change in S&P 500 index prices and \( \varepsilon_t \) is the standard error of the regression.

I tried to understand in which measure, for the period June 2004 to June 2018, with 169 monthly observations and an R-squared of 45.43%, the S&P 500 index returns can explain the VIX index; the results has been:

\[
VIX_{t_t} = 0.01840 - 3.46751S^P_X_t + 0.14981
\]

The estimated intercept, \( \alpha_0 \), is not significant different from 0, so reflects the absence of deterministic growth. This is because a property of the volatility is that is a mean-reverting process: when the VIX index is high, it tends to go down; when the VIX index is low, the contrary happens.
The estimated coefficient, $\beta_0$, is negative and significant different from 0, so reflects the inverse relationship between the dependent and the independent variables. The Figure 5 shows a representation of the negative correlation between the VIX index and the estimated VIX index explained by the only S&P 500 index variable.

Figure 3.3: The approximation diagram: tracing between the VIXr and the estimated VIXr. Historical data available on: <https://it.investing.com> [Date of access: 28/11/2018].

3.6 The VIX instruments.
The VIX index itself cannot be tradable; for this reason, there exist many instruments that can be included in the investor’s portfolio: options, futures, Exchange-Traded Funds.

VIX futures
VIX futures were introduced in 2004 by the CBOE Futures Exchange as an instrument with the purpose to hedge equity returns and to diversify the investor’s portfolio.
With respect to the VIX index, VIX futures prices can be higher or lower because future market expectations of volatility vary month to month. There is no arbitrage trading between the two instruments.

**VIX options**

VIX options were introduced in 2006 by the CBOE Options Exchange as risk management tools and as the first volatility instrument listed in a SEC-regulated securities exchange. VIX options prices are based on forward VIX value, so can be different from VIX index.

**VIX ETFs**

VIX ETFs are products conceived and managed from mutual funds companies different from the CBOE. The first product was iPath S&P 500 VIX Short-Term Futures ETN (VXX), launched in 2009; then, typical VIX ETFs are more recent because were introduced from 2011. Prices are not equal to the VIX index because funds price are subject to a mix of various components.

**3.7 The VIX properties.**

The VIX index has many properties and if the investor includes VIX instruments in the financial portfolio, may benefit of some important advantages.

**Asymmetrical correlations**

During market downturns, the rise of the VIX index is much larger than its fall during market upturns; therefore, a feature typically of the VIX index is that, in the long-term, the spikes up very fast instead the coming back happens gradually. One of the first studies done on this property of the VIX index dates back to the work of Law (2004).

**Diversification and hedging**

Considering the aspects described above, as the negative correlation between the VIX index and S&P 500 index, the most intuitively advantage is the diversification from including a long volatility position in the portfolio, more useful in market downturns. In particular, in a paper of the EDHEC-Risk Institute Publication in 2012, results state that “adding a long volatility exposure to an equity portfolio would result in a substantial improvement of the risk-adjusted performance of the portfolio.”

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Conclusions are that a portfolio diversified with long volatility instrument is a more efficient method for managing risk. Related to diversification, there is the hedging property, that helps to compensate the losses due to a bear market, by including a VIX instrument in the portfolio.

**Mean-reverting**

Another property is the mean-reverting property, thanks to which the investor can understand better the shape of the VIX term structure and can trade in response of risk changes. This property states that prices and returns come back to the long-run mean of the historical series, in this case, of the VIX index. Therefore, if the VIX index reaches a particular high level and stays far from the long-term average, after certain period, returns back to the previous normal path.

**Price range**

Usually, there exist a scale of the VIX prices range, well described in the paper of S&P Dow Jones (2017):

- if the VIX level is below 12, it is considered to be “low”;
- if the VIX level is between 12 and 20, it is considered to be “normal”;
- if the VIX level is above 20, it is considered to be “high”.

Therefore, when the VIX index is at low levels, this means that the market and investors do not expect high changes in the S&P 500 index; contrary, with high levels, the expected volatility is alarming. The VIX index could never reach the 0 because this would mean that nobody expects daily movements in the S&P 500 index.

**3.8 The VIX performance.**

The VIX started to be published, with the new methodology considering as the underlying the S&P 500 index options, by the CBOE on 22 September 2003, with an open price of 19.08 and a close price of 19.65.

The all-time highest VIX close was 80.86 on 20 November 2008, while the all-time lowest was 9.14 on 03 November 2017.

The all-time highest intraday VIX value was 89.53 on 24 October 2008, while the all-time lowest was 8.56 on 24 November 2017.
The critical periods represented in the Fig. 6 are the following:

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
<th>VIX daily change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>07 October 1997</td>
<td>Asian Financial crisis</td>
<td>-6.30</td>
</tr>
<tr>
<td>27 August 1998</td>
<td>A Russian mountain ride</td>
<td>+21.35</td>
</tr>
<tr>
<td>31 August 1998</td>
<td>Russian Financial crisis</td>
<td>+11.17</td>
</tr>
<tr>
<td>4 January 2000</td>
<td>Fear of FED rate</td>
<td>+10.94</td>
</tr>
<tr>
<td>14 April 2000</td>
<td>End of tech bubble</td>
<td>+13.03</td>
</tr>
<tr>
<td>17 September 2001</td>
<td>Aftermath of 09/11</td>
<td>+27.12</td>
</tr>
<tr>
<td>29 September 2008</td>
<td>Lehman Brothers failure</td>
<td>+29.63</td>
</tr>
<tr>
<td>02 October 2008</td>
<td>Weak and frozen credit market</td>
<td>+12.83</td>
</tr>
<tr>
<td>06 October 2008</td>
<td>Bailouts doubts begin</td>
<td>+14.24</td>
</tr>
<tr>
<td>20 May 2010</td>
<td>European Debt crisis and falling Euro</td>
<td>+25.96</td>
</tr>
<tr>
<td>10 August 2011</td>
<td>Concern about European Debt crisis</td>
<td>+20.39</td>
</tr>
<tr>
<td>09 November 2011</td>
<td>Italian crisis, 10-year bond yield up 7%</td>
<td>+27.45</td>
</tr>
<tr>
<td>24 June 2016</td>
<td>Day after Brexit</td>
<td>+40.10</td>
</tr>
<tr>
<td>09 August 2017</td>
<td>Slower U.S. economy, lingering problems in Asia, slimmer corporate problems</td>
<td>+1.36</td>
</tr>
</tbody>
</table>

Table 1.1: The VIX index daily percentage change in critical periods.
3.9 The VIX studies.

Many studies are conducted on the potential diversification advantages of the VIX index. The developer of the VIX index, Robert E. Whaley (2008) explains that it is a forward-looking measure of volatility and that the two initial purposes of the index are: the first is that VIX index was intended to offer a benchmark of expected short-term volatility of the market; the second is that VIX index was intended to offer an index upon which futures and options contracts could be created.

In 2004, a paper of the Credit Suisse First Boston Ltd. (CSFB) notes, in particular, that the volatility tends to be mean-reverting and to cluster.

In 2005, Dash and Moran found that introducing a portion of VIX, from 0% to 10%, is a benefit tool also for portfolios containing alternative investments. In particular, they found it is negatively correlated with hedge fund returns and this correlation is asymmetric with more negative coefficient in negative months of hedge funds.

In 2006, Daigler and Rossi confirm that including a long exposure of VIX index in a S&P 500 stock portfolio, can allow to diversification with particular benefits in terms of reducing the risk. In practical, this can be possible by purchasing, not only futures and options, but also ETFs on volatility.

In 2006, Black found that the addition of a small VIX exposure in an investment portfolio can reduce the portfolio volatility and the skewness and the excess kurtosis of many hedge funds. In 2007, Grant, Gregory and Lui, belonging to Goldman Sachs, have written two papers: one in which they compared VIX call options and SPX put options and concluded that VIX call options can effectively diversify the equity risk. The other one in which selectively long VIX exposure can lead to diversification benefits, including the decision of short the volatility (that they consider being an asset class).

One of the first works carried out with the objective to understand if the VIX index can be a benefit tool for financial portfolios and with which weight assign to this element, is that explained by Dash and Moran in “VIX as a Companion for Hedge Fund Portfolios” contained in the Winter 2005 edition of the Journal of Alternative Investments; they consider hedge funds since are the most aggressively managed funds, subject to high volatility.

In particular, in this work the authors propose to explore the relationship between the VIX index and hedge fund returns, through the observation of the correlation; then to test if the allocation of the VIX index in a portfolio composed only by an hedge fund index, the Credit Suisse Hedge Fund Index, in the period included between 1995 and 2004, can reduce risk and provide downside protection to investment, comparing portfolio with and without the VIX index.
After ascertaining that the correlation is asymmetric, due to the fact that when market goes down, the correlation between the VIX index and the hedge fund index is strong with respect to the correlation when the market goes up, they look at the efficient frontier and try to make the best allocation of the VIX index.

They study 3 portfolios with 3 different strategies:
1. hedge fund index with no VIX index;
2. hedge fund index with a static allocation of 5% of the VIX index;
3. hedge fund index with a tactical allocation of the VIX index that changes on the basis of the change in the VIX index of the past quarter: 0% if the VIX index has increased by more than 20%; 10% if the VIX index has decreased by more than 20%; 5% if the change has been between -20% and +20%.

Comparing different indicators, among which returns, volatilities, Sharpe ratios, based on the mean-reverting property, they stated that the best way to allocate the VIX index in this type of portfolio is the tactical strategy.

Another study among the most interesting and practical papers, that include all the previous ideas and is based primarily on the work of Dash and Moran (2005), is “VIX Futures and Options – A Case Study of Portfolio Diversification During the 2008 Financial Crisis” written by Edward Szado in 2009. The study evaluates the impact of long VIX instrument, as a diversifier tool, in a portfolio constructed in the period from March 2006 to December 2008. The core of the paper considers the fact that: “VIX may spike upwards as the S&P 500 experiences large drops, leading one to believe that a long VIX position could provide significant diversification benefits to an equity portfolio.”\(^{11}\). For achieving the purpose of the paper, the author considers three different portfolios, with different asset allocations, and adds a long exposure of VIX futures and options to see if really the VIX could help the portfolio to be diversified during the financial crisis of 2008, also considering and comparing the common protection strategy for equity through hold put options, that is most expensive.

The investment is the VIX index is as follows: “a long position in the front-month VIX futures contract is fully collateralized by holding the full value of the contract in Treasury bills”\(^{12}\). It is introduced in these portfolios:

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\(^{12}\) Ivi, p. 9.
1. 100% Stocks;
2. 60% Stocks/40% Bonds;
3. 60,5% Stocks/30,5% Bonds/1,3% High Yield Bonds/1,2% Hedge Funds/0,1% Managed Futures/0,3% Commodity/1,6% Private Equity/4,5% Real Estate).

The VIX allocation occurs with both futures and options.

The author found that, adding a 2,5% allocation to the VIX futures, enhanced returns and reduces the standard deviations. And making comparison with a portfolio without VIX allocation and a portfolio with the addition of alternative assets (as portfolio no. 3), in the latter, diversification does not work in the crisis period.

He found also that with addition of VIX call options the returns improved while standard deviations decrease in a restrained manner.

Therefore, in all tests, the adding of the VIX index improves the risk/return issue and protects the portfolio in the turbulence period of the financial crisis and the VIX call options, in particular, could lead to more efficient diversification with respect to the SPX put options.
Part 2 – The practical application: a real financial portfolio.

In the Part 2, I chose mutual funds for a specific investor and created a financial portfolio. To pursue this goal, first of all, I chose the mutual funds that reflect the most important selection of asset classes, mainly with the support of the Morningstar website; in particular, I choose to work with monthly NAV, found on the Thomson Reuters Eikon software and Investing.com website, for a more convenient management of values. Then, I give weights to each mutual fund, looking at the historical values of the previous 4 years, having as reference the Markowitz portfolio theory, through the construction of the efficient frontier in the risk-return spectrum.

With the support of software applications (Microsoft Excel, R), I monitored the portfolio, with respect to different indicators, in a predetermined period, to see how it behaved. Cost implications are not considered, since have a high degree of variability that depends on the advisor and the financial institution that hold the current account of the investor and make operations on his behalf.

Finally, I implement two strategies: “Buy and hold” and tactical allocation strategies, the latter divided in static allocation and ex-post allocation, practically, to understand if the studies, reported in the Chapter 3 of the Part 1 of this work, can be put into practice in the reality.

The Part 2 is composed by six chapters:

- Chapter 1. Characteristics of the investor and market conditions.
- Chapter 2. The financial portfolio realization.
- Chapter 4. Characteristics and performance of the portfolio including the VIX instrument.
- Chapter 5. The “Buy and hold” and 1-year tactical strategies.
Chapter 1. CHARACTERISTICS OF THE INVESTOR AND MARKET CONDITIONS.

Before constructing a financial portfolio, there are two steps to follow: the first is to fix peculiarities and aspects of the principal subject that has the interest: the investor; the second is to analyze and catch the market trend through historical values.

1.1 Characteristics of the investor.

The investor is an engineer working in the oil field, 38 years old, who lives in Italy. He has the following characteristics:

- he is risk-averse in the meaning of the Modern Portfolio Theory, that is, prefers a portfolio with the highest expected return for a given level of risk or with the lowest variance for a given level of return; but, at the same time, he is risk-lover since is inclined to innovations and to foreign investments, so has a capacity of high risk tolerance and high risk capacity;
- the available amount, € 80,000,00, has the only objective to be invested, it is not available in terms of investor’s liquidity purposes, therefore, he does not want to redeem it. The amount of € 20,000,00, is left in the current account for liquidity and rebalancing purposes;
- he has a long-term liquidity horizon (e.g. more than 10 years);
- he does not expect to receive a prefixed percentage of returns;
- he prefers using mutual funds to be sure of trading the NAV at the close of any trading day.

1.2 Conditions of the market.

The investor decides to employ the money at the end of 2006; in that period, the market was relatively calm even if, after the financial burst in 2007-2008, we will know that the subprime mortgage crisis was already in action at the end of 2006 and the United States housing bubble has its peak in 2005-2006.

Annual average exchange-rate EUR/USD was relatively stable from 2004 at a rate of 1.24 to 1.26 in 2006.

From 2006, it was coined for the first time the term E7, that is the acronym for a group of emerging markets in a growing trend, composed by China, Brazil, Mexico, Russia, Indonesia and Turkey.
Chapter 2. THE FINANCIAL PORTFOLIO REALIZATION.

Considering the previous statements, at the beginning of January 2007, I choose mutual funds with the support of Morningstar website\(^\text{13}\) and Thomson Reuters Eikon software\(^\text{14}\). A common characteristic to these funds is that everyone belongs to a class of accumulation shares; this means that all income earned is kept in the price of the shares, i.e. dividends are reinvested automatically into gains generated.

2.1 Description of mutual funds.

The mutual funds and their objectives are the following:

- **Janus Henderson - Balanced A EUR Acc (Hedged):** it focuses on investments in equity securities. It invests a certain percentage of its value in the shares of financial companies and the remaining percentage in the debt securities of US issuers (companies established or operating in the United States or that derive a portion of the property or profit rights from this area) and loans.

- **Pictet - Digital R in EUR:** it invests primarily in shares of companies offering digital products or services, including services that enable digital interactivity. It invests worldwide, including emerging markets and Mainland China.

- **Pictet - Water R EUR:** it invests primarily in shares in water distribution or treatment companies, water technology or environmental services. It invests worldwide, including emerging markets and Mainland China.

- **Goldman Sachs (GS) - Global High Yield Pf E Acc EUR:** It invests primarily in below investment grade bonds issued by companies that are based in or make most of their profits or revenues in North America and/or in Europe.

- **BlackRock Global Funds (BGF) - Emerging Markets A2 EUR:** it invests globally mainly in equity securities of companies domiciled or exercising the predominant part of their economic activity in emerging markets. The same type of investment may also be made in developed markets that have significant business operations in emerging markets.

- **Carmignac Pf Commodities A EUR Acc:** it invests in the entire sector of natural resources (energy, precious metals, base metals, agricultural raw materials and timber).

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\(^{13}\) Morningstar is a Chicago-based independent investment research firm that through its website offers information about a wide range of funds. The website is available on: \(<\text{http://www.morningstar.it}>\).

\(^{14}\) Eikon is a software, launched in 2010 by Thomson Reuters, a Canadian multinational mass media and information firm, that provides financial information and data.
• Banque de Luxembourg investment funds (BL) – Equities Europe B EUR Acc: it invests mainly in equity securities of companies located in a member country of the European Union. The remaining part of the portfolio is invested in companies listed on a regulated European market.

• Schroder International Selection Fund (SISF) – EURO Government Bond C Acc EUR: it invests in bonds and other fixed and floating rate securities issued by European governments, government agencies, supranational bodies and corporate issuers.

Other detailed characteristics are showed in the Table 2.1:

• ISIN (International Securities Identification Number), the code that needs to identify the mutual fund; the initial two characters identify the country where the value is quoted: in my case, IE is Ireland, LU is Luxembourg.

• Morningstar category\footnote{Morningstar category list is available on: \url{http://www.morningstar.com/InvGlossary/morningstar_category.aspx} [Date of access: 28/11/2018].}, that is not the object of the mutual fund, but described in few words on which underlying securities is based.

• Starting date, represents the date from which the quotation started.

<table>
<thead>
<tr>
<th>Name</th>
<th>ISIN</th>
<th>Morningstar category</th>
<th>Starting date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Janus Henderson Balanced A EUR Acc (Hedged)</td>
<td>IE0009514989</td>
<td>Balanced Moderates - Global EUR</td>
<td>31/12/1999</td>
</tr>
<tr>
<td>Pictet-Digital R USD</td>
<td>LU0101692753</td>
<td>Equities - Technology</td>
<td>26/11/1999</td>
</tr>
<tr>
<td>Pictet-Water R EUD</td>
<td>LU0104885248</td>
<td>Equities - Water</td>
<td>29/02/2000</td>
</tr>
<tr>
<td>GS Global High Yield Pf E Acc EUR</td>
<td>LU0133266659</td>
<td>Bonds - High Yield Global</td>
<td>31/07/2001</td>
</tr>
<tr>
<td>Carmignac Pf Commodities A EUR Acc</td>
<td>LU0164455502</td>
<td>Equities - Natural Resources</td>
<td>10/03/2003</td>
</tr>
<tr>
<td>BL Equities Europe B EUR Acc</td>
<td>LU0093570330</td>
<td>Equities - Europe Large Cap Growth</td>
<td>28/09/1990</td>
</tr>
<tr>
<td>SISF EURO Government Bond C Acc EUR</td>
<td>LU0106236184</td>
<td>Bonds - Government EUR</td>
<td>17/01/2000</td>
</tr>
</tbody>
</table>

Table 2.1: Mutual funds characteristics. Information available on <http://www.morningstar.it/it/> [Date of access: 28/11/2018].

\footnotetext[15]{Morningstar category list is available on: \url{http://www.morningstar.com/InvGlossary/morningstar_category.aspx} [Date of access: 28/11/2018].}
2.2 Portfolio historical values: January 2003 – December 2006.

The choice of the previous mutual funds is based, technically, on historical characteristics, belonging to the period January 2003 – December 2006. Sources from which NAVs are taken are Thomson Reuters Eikon software and Investing.com website\(^\text{16}\).

**Historical NAV**

The Figure 2.1 represents the historical NAV of each mutual funds; the x axis represents dates from January 2003 to December 2006, instead, the y axis represents the NAV: in particular the left y axis is the reference for all the mutual funds, except for the BL – Equities Europe B EUR Acc, that has the right side of the y axis as reference; this difference for a clearer representation.

It can be observed a positive trend for Pictet-Digital-R USD, Pictet-Water-R EUR, Carmignac Pfl Commodities A EUR Acc and BL – Equities Europe B EUR Acc.

![Figure 2.1: Historical NAV Jan 2003 – Dec 2006.](image)

**Historical returns**

The Figure 2.2 represents historical returns of each mutual funds; the x axis represents dates from January 2003 to December 2006, instead, the y axis represents returns.

It can be observed a trend quite around the zero, except for some spikes: +36,967% for GS Global High Yield Pf E Cap EUR in April 2004; -13,612% for Carmignac Pfl Commodities A EUR Acc in the same month of 2004. A little more deviation for BGF Emerging Markets A2 EUR and Carmignac Pfl Commodities A EUR Acc.

\(^{16}\) Investing.com is a global financial portal that provides news, tools, data about the global financial market. Available on: [https://www.investing.com](https://www.investing.com) [Date of access: 28/11/2018].
Figure 2.2: Historical returns Jan 2003 – Dec 2006.

**Mean and Standard deviation**

In the bar chart, Figure 2.3, the mean and the standard deviation for each mutual fund are represented; the x axis represents mutual funds, the y axis represents mean and standard deviation in percentage terms.

It can be observed a mean of 1.292% on average and a standard deviation of 3.591% on average. The highest mean belongs to Carmignac Pfl Commodities A EUR Acc; the lowest standard deviation belongs to SISF – EURO Government Bond C Acc EUR.

Figure 2.3: Mean and Standard deviation.
Correlation matrix

The Table 2.2 represents the correlation between each mutual fund with respect to each other. It can be observed more correlation between the equity mutual funds; GS Global High Yield Pf E Cap EUR and SISF EURO Government Bond C Acc EUR, instead, are the diversifier mutual funds since have lower correlation with respect to the other mutual funds.

Table 2.2: Correlation matrix.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SISF EURO Government Bond C Acc EUR</td>
<td>0.061</td>
<td>-0.066</td>
<td>-0.019</td>
<td>0.013</td>
<td>-0.284</td>
<td>0.033</td>
<td>-0.154</td>
<td>1.00</td>
</tr>
<tr>
<td>BI-Equities Europe B EUR Acc</td>
<td>0.535</td>
<td>0.698</td>
<td>0.762</td>
<td>0.762</td>
<td>0.339</td>
<td>0.377</td>
<td>1.00</td>
<td>-0.154</td>
</tr>
<tr>
<td>Carmignac Pfl Commodities A EUR Acc</td>
<td>0.483</td>
<td>0.254</td>
<td>0.310</td>
<td>0.581</td>
<td>-0.366</td>
<td>1.000</td>
<td>0.377</td>
<td>0.033</td>
</tr>
<tr>
<td>Goldman Sachs Glo High Yield Pf E Cap EUR</td>
<td>-0.195</td>
<td>0.066</td>
<td>0.109</td>
<td>-0.001</td>
<td>1.000</td>
<td>-0.366</td>
<td>0.339</td>
<td>-0.284</td>
</tr>
<tr>
<td>BGF Emerging Markets A2 EUR</td>
<td>0.552</td>
<td>0.666</td>
<td>0.700</td>
<td>1.000</td>
<td>-0.001</td>
<td>0.581</td>
<td>0.762</td>
<td>0.013</td>
</tr>
<tr>
<td>Pictet-Water-R EUR Open Fund</td>
<td>0.585</td>
<td>0.736</td>
<td>1.000</td>
<td>0.700</td>
<td>0.109</td>
<td>0.310</td>
<td>0.762</td>
<td>-0.019</td>
</tr>
<tr>
<td>Pictet-Digital-R USD</td>
<td>0.591</td>
<td>1.000</td>
<td>0.736</td>
<td>0.666</td>
<td>0.066</td>
<td>0.254</td>
<td>0.698</td>
<td>-0.066</td>
</tr>
<tr>
<td>Janus Henderson Balanced A EUR Acc (Hedged)</td>
<td>1.000</td>
<td>0.591</td>
<td>0.585</td>
<td>0.552</td>
<td>-0.185</td>
<td>0.483</td>
<td>0.535</td>
<td>0.061</td>
</tr>
</tbody>
</table>

Asset allocation

The asset allocation chosen, considering the investor characteristics and the historical values of mutual funds, can be viewed from different perspectives: asset classes, sectors, geographical areas.

Asset class

The pie chart in Figure 2.4 represents the asset allocation of each mutual funds viewed in the perspective of asset classes. Investing in equal parts in all mutual funds chosen, means having 64.47% invested in the equity asset class, 29.88% in the bond, 5.35% in the money and a 0.32% in other.
This representation reflects the risk profile and the time horizon preferences of the investor: he is risk-lover and has a long-time horizon, therefore, equity asset class is weighted more than other asset classes. Then, bond and monetary asset class try to decrease the total risk of the portfolio.

Figure 2.4: Asset allocation: asset class.

**Sector**

The pie chart in Figure 2.5 represents the asset allocation of equity mutual funds viewed in the perspective of sectors. The most invested sector is the technological one; then industrial goods and raw materials.

The sector of technology, weighted 20.47%, is in a developing path after the Dot-com bubble developed in the period 1997-2000.

The sector of industrial goods, weighted 15.23%, represents the basis for all the business activity.

The sector of raw materials, weighted 10.93%, represents the basis for the production activity.

Figure 2.5: Asset allocation: sector.
**Geographical area**

The pie chart in Figure 2.6 represents the asset allocation of equity mutual funds viewed in the perspective of geographical areas. The most invested geographical area is United States of America; then Western Europe and Emerging Asia. The USA is the most weighted country, 43.88%, since in the history has always been the most developed, liquid and efficient financial market and assets have been always the most attractive, also for the consistent return offered. The Western European area is weighted 13.71% since is the geographical area in which the investor works and lives, is the most known market by the investor, therefore, is a sort of protection for his investment. The Asia Emerging is weighted 11.30% since is the part of the portfolio, together with the other developing countries, that gives more volatility but also more potential return.

![Asset Allocation: geographical area](image.png)

Figure 2.6: Asset allocation: geographical area.

**2.3 The asset allocation of the investor’s portfolio.**

The pie chart, in Figure 2.7, represents the weights given to each mutual fund, within the investor’s portfolio, established at the beginning of January 2007. The weights are given on the basis of all issues described above in this Chapter and from the implementation of the Markowitz theory through the construction of the efficient frontier for the period January 2003 – December 2006, represented in Figure 2.8.

In the scatterplot, in Figure 2.8, it can be observed that the portfolio of the investor is placed on the right-hand side with respect to the efficient frontier. The portfolio in 3 years, from January 2003 to December 2006, has performed a return of 1.375% and a standard deviation of 8.071%, better than an equally weighted portfolio but less efficient than the Sharpe portfolio.
The portfolio is not on the efficient frontier because the weights resulting for each mutual fund are subjected to two main constraints:

- for the principle of the inclusion of all mutual funds in the portfolio, each mutual fund must be in the portfolio for a weight at least of 1%;
- for the principle of non-concentration, each mutual fund cannot exceed the weight of 25%.

The resulting asset allocation, considering the constraints, makes that the portfolio does not lie on the efficient frontier but is the more efficient among not efficient portfolios.

Figure 2.7: Asset class: mutual fund.

Figure 2.8: Efficient Frontier Jan 2003 – Dec 2006.

At this point, my analysis carries on, shifting the attention on a longer period: from January 2007 to June 2018, for understanding in which way the portfolio, composed at the beginning of the 2007, expresses itself in a period full of unexpected events, among which the financial crisis of 2007-2008, European debt crisis with its peak in 2010-2012, Brexit in 2016. This analysis starts similarly to the analysis done prior for the choice of mutual funds and then follows with different indicators that help to understand even better in which way the performance has been in this long period. The portfolio management policy used is the passive strategic policy, called also “buy and hold”, since does not requires any trading activities.

3.1 Portfolio historical values.

In this section, it can be observed how really the investor’s portfolio has performed from January 2007 to June 2018, starting from the NAV of each mutual fund, then returns and related mean and standard deviation and the correlation matrix.

**Historical NAV**

The line graph in Figure 3.1 represents the historical NAV of the mutual funds selected, in the period of January 2007 – June 2018. The x axis represents the monthly dates, instead, the y axis represents the NAV: in particular the left y axis is the reference for all the mutual funds, except for the BL – Equities Europe B EUR Acc, that has the right side of the y axis as reference.

It can be observed a positive trend for Pictet-Digital-R USD, Carmignac Pfl Commodities A EUR Acc and BL-Equities Europe B EUR Acc; even if, the commodities fund has experienced large up and down moves.

![Figure 3.1: NAV Jan 2007 – Jun 2018.](image-url)
Historical returns

The line graph in Figure 3.2 represents returns of mutual funds selected for the period January 2007 – June 2018.

In October 2008, all mutual funds, except the European government bonds, have experienced a large drop, concurrently, with the worst financial crisis of the millennium. The mutual fund that lost more has been the commodities one, followed by the high yield one.

In September 2011, European equity and commodities have experienced an important drop.

In August 2015, primary, High Yield and European equity have experienced a quite big drop. Similarly, in January 2016, High Yield and commodities.

There have been also months of upwards moves. For example, in August 2007, for High yield and commodities; in April 2009, for the same mutual funds; in December 2015, for Digital, European equity and commodities.

![Figure 3.2: Returns Jan 2007 – Jun 2018.](image)

Mean and Standard deviation

The bar chart in Figure 3.3 shows the mean and the standard deviation for each mutual fund; the x axis represents mutual funds, the y axis represents mean and standard deviation in percentage terms.
I can observe a portfolio mean of 0.420% on average and a portfolio standard deviation of 3.847% on average. The highest mean belongs to Pictet-Digital-R USD; the lowest standard deviation belongs to SISF – EURO Government Bond C Acc EUR.

Figure 3.3: Mean and Standard deviation.

**Correlation matrix**

The Table 3.1 represents the correlation between each mutual fund with respect each other in the period January 2007 – June 2018.

With respect to the period between 2003-2006, before the financial crisis, correlations between asset classes, in general, are increased. The only exception is for the Euro Government Bond asset class; it has more negative correlation with respect to all other mutual funds. It is the sole instrument that can bring diversification.

<table>
<thead>
<tr>
<th>Mutual fund</th>
<th>-0.146</th>
<th>-0.023</th>
<th>-0.012</th>
<th>0.045</th>
<th>-0.087</th>
<th>-0.237</th>
<th>-0.016</th>
<th>1.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>SISF - EURO Government Bond C Acc EUR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BI-Equities Europe B EUR Acc</td>
<td>0.737</td>
<td>0.744</td>
<td>0.843</td>
<td>0.522</td>
<td>0.797</td>
<td>0.593</td>
<td>1.000</td>
<td>-0.016</td>
</tr>
<tr>
<td>Carmignac Pfl Commodities A EUR Acc</td>
<td>0.667</td>
<td>0.563</td>
<td>0.574</td>
<td>0.348</td>
<td>0.780</td>
<td>1.000</td>
<td>0.593</td>
<td>-0.237</td>
</tr>
<tr>
<td>GS - Glo High Yield Pf E Cap EUR</td>
<td>0.728</td>
<td>0.321</td>
<td>0.746</td>
<td>0.433</td>
<td>1.000</td>
<td>0.780</td>
<td>0.757</td>
<td>-0.087</td>
</tr>
<tr>
<td>BGF - Emerging Markets A2 EUR</td>
<td>0.693</td>
<td>0.557</td>
<td>0.649</td>
<td>1.000</td>
<td>0.433</td>
<td>0.348</td>
<td>0.522</td>
<td>0.045</td>
</tr>
<tr>
<td>Pictet-Water-R EUR</td>
<td>0.623</td>
<td>0.786</td>
<td>1.000</td>
<td>0.639</td>
<td>0.736</td>
<td>0.574</td>
<td>0.823</td>
<td>-0.012</td>
</tr>
<tr>
<td>Pictet-Digital-R USD</td>
<td>0.623</td>
<td>1.000</td>
<td>0.786</td>
<td>0.557</td>
<td>0.721</td>
<td>0.563</td>
<td>0.744</td>
<td>-0.023</td>
</tr>
<tr>
<td>Janus Henderson - Balanced A EUR Acc (Hedged)</td>
<td>1.000</td>
<td>0.623</td>
<td>0.625</td>
<td>0.193</td>
<td>0.728</td>
<td>0.697</td>
<td>0.737</td>
<td>-0.146</td>
</tr>
</tbody>
</table>

Table 3.1: Correlation matrix.
3.2 The analysis of the portfolio.

At this point, I analyze how the entire portfolio behaved, in terms of risk and return, from its starting date until the first half of the 2018, that is for 11 years and 6 months and 137 monthly observations in total.

3.2.1 The portfolio and the efficient frontier.

The portfolio, with the same weights given at the beginning of 2007, represented in Figure 3.4, shows a very low return, 0.420%, related to a substantial risk, 10.506%. It is very far from the most efficient portfolio, the Sharpe one, and from the entire efficient frontier.

In Figure 3.5 the cumulative returns of the portfolio is represented; it remains most of the time between about -6% to +5% but experiences also large losses that are not recovered.

Figure 3.4: Efficient Frontier Jan 2007 – Jun 2018.

Figure 3.5: Portfolio Returns.
3.2.2 Fundamental components of the Modern Portfolio Theory.

For computing asset pricing models that allow to understand in which way the market and the safest asset explain the return of each element of the portfolio, I had to set two elements: the benchmark and the risk-free asset.

**The benchmark: the MSCI World index**

I chose the MSCI World index as benchmark, that is as a representative tool for the market, because well-explains the investor’s portfolio of this work in terms of asset allocation in the perspective of sector and geographical area.

**The risk-free asset: the Carmignac Court Terme A EUR Acc**

I chose the Carmignac Court Terme A EUR Acc mutual fund as risk-free asset, because it is a short-term monetary fund that invests in bonds issued by high-rated issuers and has as objective to preserve the invested capital, remunerating close to the money market rate.

**Asset pricing models**

In the CAPM and Fama & French 5-factor models, I investigated the contribution, of different variables, on the return of the mutual funds of the investor’s portfolio. For the CAPM model, the main variable is the excess return between the benchmark and the risk-free rate; for the Fama & French 5-factor models, besides the excess return between the benchmark and the risk-free rate, there are other four variables recovered from data library of the website of Kenneth R. French17, one of the authors of the model. F&F 5-factor model, in this work, needs only to know the tilts of mutual funds of the portfolio towards each of 5 factors.

The results for the portfolio in Table 3.2, in the period January 2007 – June 2018, show that the $\alpha$, the ability to generate value with respect to the benchmark, of the portfolio under the CAPM model is 0,00334; under F&F 5-factor model is 0,00196. The $\beta$, the measure of the systematic risk, of the portfolio under the CAPM model is 0,50083; under F&F 5-factor model is 0,50077. These betas, highly close, mean that the portfolio has, almost, the half volatility and return with respect to the market.

---

Regarding the F&F 5-factor model, it can be observed that the entire portfolio has a positive exposure to small stocks, negative exposure to other variables: the only two significant parameters are $\beta$ and SMB that is 0.18560; this latter means that the portfolio is predominantly towards small capitalization stocks with respect to large ones.

<table>
<thead>
<tr>
<th>Model/variable</th>
<th>CAPM</th>
<th>F&amp;F 5-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coefficient</td>
<td>p-value</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.00334</td>
<td>0.04988</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.50083</td>
<td>0.00000</td>
</tr>
<tr>
<td>SMB</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>HML</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>RMW</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>CMA</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.58199</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Asset pricing models: results.

**Downside measures of risk**

For what concerns the downside measures of risk, the investor’s portfolio, both measures, VaR and ES, are calculated annually and with a 95% level of confidence on the basis of the historical methodology, that is, are calculated referring to the historical database of components of the portfolio; even in the same period, behaved in the following way:

- the 5% VaR of the portfolio, showed in Figure 3.5 with bars blue for each mutual fund, is equal to -4.334%;
- the 5% ES of the portfolio, showed in Figure 3.5 with bars orange for each mutual fund, is equal to -5.435%;

![Figure 3.6: Value at Risk vs. Expected Shortfall (5%).](image)
The Maximum Drawdown of the portfolio, in the “buy and hold” portfolio management policy, is -39.48%; in the Figure 3.6, the drawdowns for each mutual fund are represented.

![Figure 3.7: Drawdown Jan 2007 – Jun 2018.](image)

### 3.2.3 Conclusion.

The investor's portfolio, at the end of the period considered, held with a passive management policy, “buy and hold”, has performed very poorly in front of a high volatility; this result does not agree with the objectives set by the investor.

Then, in the period January 2007 to June 2018, it can be observed that correlation matrix differs importantly with respect to the correlation matrix for the period January 2003 to December 2006; the explanation is that, in the longer period considered, many turbulent events have happened. Especially, mutual funds included in the investor’s portfolio have experienced large downfalls from the end of 2007 to the end of 2008, with the worst drop in October 2008, corresponding to the subprime financial crisis; in 2011, corresponding to the European sovereign debt crisis; in 2015, corresponding to a series of facts: “Grexit” risk, terrorist attacks in France, Chinese stock market turbulence, the rise of USA interest rate by the Fed.

Even if, the investor’s portfolio is relatively diversified in terms of quantity, since are present 8 mutual funds, and in terms of quality, considering various asset classes, sectors, geographical areas present, correlations between mutual funds are not so good in terms of diversification. In fact, the only instrument that could bring a certain degree of diversification is the SISF – EURO Government Bond C Acc EUR mutual fund, being weakly or, in some cases, negatively correlated with respect to the other mutual funds.
Therefore, in periods in which the market experienced downturns, all the major asset classes tend to be highly correlated among them and, considering the correlation analysis, only the Euro government bond asset class can guarantee a minimum level of protection in the portfolio.

From these considerations, derives the fact of trying to improve the investor’s portfolio. Concretely, it means to try to increase its return and decrease its risk, to diversify in a better way the asset classes already included in it and to enhance the portfolio hedging in cases of market downturns.

To achieve these purposes, in the Chapter 4 of this part, I thought to include, within the portfolio, a tool that refers to the CBOE Volatility index: the VIX.
Chapter 4. CHARACTERISTICS AND PERFORMANCE OF THE PORTFOLIO INCLUDING THE VIX INSTRUMENT.

In this Chapter, I observe how the behavior of the investor’s portfolio would have been, with the investment of his money also in the VIX index. The idea is realized based on the study of Dash and Moran (2005), explained in Part 1 of this work.

This instrument is constructed with the combination of two components for reasons of period of analysis; it is explained in the Section 4.1.

Then, in the Section 4.2, the analysis of the portfolio including the VIX instrument follows.

4.1 The VIX instrument components: the underlying index and the ETF.

Since the VIX index is not tradable, I found a valid substitute to include in the investor’s portfolio in the “Lyxor S&P 500 VIX Futures Enhanced Roll UCITS ETF - Acc”. But, in the practical case, since this fund was established on 25 September 2012, and my portfolio case study starts from January 2007, I extended the fund backwards, using as proxy the underlying index replicated from the ETF: the S&P 500 VIX Futures Roll Enhanced TR index.

4.1.1 The S&P 500 VIX Futures Roll Enhanced TR.

The S&P 500 VIX Futures Roll Enhanced TR is an index that matches dynamically a short-term VIX futures portfolio and a medium-term one for managing in an efficient way the exposure to the stock volatility market.

The short-term portfolio is represented by the S&P 500 VIX Short-Term Futures index, an index that measures the return from a rolling long position in two VIX futures contracts with adjacent maturities. In practice, the index rolls continuously each month from the shorter-term VIX futures contract to the longer-term one.

The medium-term portfolio manages a rolling daily position relative to the third, fourth and fifth future contract of the month.

Movement from one portfolio to the other is defined by signals based on the value of the VIX in relation to its moving average. The rebalancing becomes, generally, on a quarterly basis.

4.1.2 The Lyxor S&P 500 VIX Futures Enhanced Roll UCITS ETF - Acc.

The Lyxor S&P 500 VIX Futures Enhanced Roll UCITS ETF – Acc, replicates, indirectly or synthetically, the S&P 500 VIX Futures Roll Enhanced TR. This means that the index is replicated with a swap transaction (total return swap); the ETF enters into a contract with a financial institution, typically the parent company of the ETF issuer, which is obliged to deliver the index return, including dividend payments, in exchange for a fee. This allows to a wide range of investors to have access to particular asset classes.
The UCITS term in the name of the ETF, that stands for Undertakings for Collective Investments in Transferable Securities, means that it is compliant with a single European Union regulatory framework, that is related to the saver’s protection from unsuitable investment vehicles.

4.2 The investor’s portfolio analysis with the VIX instrument.

The line graph in Figure 4.1, represents the monthly historical NAV of the VIX instrument in the period of January 2007 – June 2018. The x axis represents the monthly dates; the y axis represents the monthly NAV.

It can be observed high level of NAV at the end of 2008, in the second half of the 2010 and at the end of 2011 to the first half of 2012, corresponding to the major periods of high volatility in the financial market.

![Figure 4.1: S&P 500 VIX Futures Enh Roll TR (Aug 2012) – Lyxor S&P 500 VIX Futures Enh Roll C EUR.](image)

**Historical returns**

The line graph in Figure 4.2, represents the historical returns of the entire portfolio, including the VIX instruments, for the period January 2007 – June 2018.

The most significant aspect that emerges in the graph, is the particular trend of the VIX instrument. It experienced large upwards moves when the mutual funds, that compose the investor’s portfolio, experienced downwards moves; this can be explained by the correlation between the VIX instrument and all other components of the portfolio.

The only exception easily observable in the graph, is the SISF – EURO Government Bond C Acc EUR mutual fund that behaves, in a lesser extent, as the VIX instrument, since its trend is almost constantly around zero and does not suffer from large downturns, even in turbulent
periods. In fact, as can be observed in the correlation matrix in Table 4.1, the unique positive correlation of the VIX instrument with respect the other elements of the portfolio, is with the European government bond mutual fund.

Figure 4.2: Historical returns: Jan 2007 – Jun 2018.

**Mean and Standard deviation**

The bar chart in Figure 4.3 shows the mean and the standard deviation for the VIX instrument; the x axis represents the VIX instrument, the y axis represents mean and standard deviation in percentage terms.

It can be observed a mean of -1.614% and a standard deviation of 13.988%, during the entire period from January 2007 to June 2018.

Figure 4.3: Mean and Standard deviation.
Correlation matrix

In the correlation matrix in the Table 4.1, the VIX instrument is negatively correlated, even consistently, with 7 out of 8 mutual funds. This means that, as the theory explains, when mutual funds experienced downward moves, the VIX instrument raises and the contrary happens. The only exception is the correlation result between the VIX instrument and the SISF – EURO Government Bond C Acc EUR; it is weakly positive.

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>VIX instrument</td>
<td>-0.743</td>
<td>-0.429</td>
<td>-0.440</td>
<td>-0.168</td>
<td>-0.554</td>
<td>-0.523</td>
<td>-0.597</td>
<td>0.175</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SISF - EURO Government Bond C Acc EUR</td>
<td>-0.146</td>
<td>-0.023</td>
<td>-0.012</td>
<td>0.045</td>
<td>-0.087</td>
<td>-0.237</td>
<td>-0.016</td>
<td>1.000</td>
<td>-0.175</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BL-Equities Europe B EUR Acc</td>
<td>0.737</td>
<td>0.744</td>
<td>0.823</td>
<td>0.522</td>
<td>0.757</td>
<td>0.593</td>
<td>1.000</td>
<td>-0.016</td>
<td>-0.597</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carmignac Pfl Commodities A EUR Acc</td>
<td>0.657</td>
<td>0.563</td>
<td>0.574</td>
<td>0.348</td>
<td>0.780</td>
<td>1.000</td>
<td>0.593</td>
<td>-0.237</td>
<td>-0.523</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GS - Glo High Yield PTF-E Cap EUR</td>
<td>0.728</td>
<td>0.721</td>
<td>0.736</td>
<td>0.433</td>
<td>1.000</td>
<td>0.780</td>
<td>0.757</td>
<td>-0.087</td>
<td>-0.554</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Janus Henderson - Balanced A EUR Acc (Hedged)</td>
<td>0.193</td>
<td>0.557</td>
<td>0.639</td>
<td>1.000</td>
<td>0.433</td>
<td>0.348</td>
<td>0.522</td>
<td>0.045</td>
<td>-0.168</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Pictet-Water-R EUR</td>
<td>0.625</td>
<td>0.786</td>
<td>1.000</td>
<td>0.639</td>
<td>0.736</td>
<td>0.574</td>
<td>0.823</td>
<td>-0.012</td>
<td>-0.440</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pictet-Digital-R USD</td>
<td>0.623</td>
<td>1.000</td>
<td>0.786</td>
<td>0.557</td>
<td>0.721</td>
<td>0.563</td>
<td>0.744</td>
<td>-0.023</td>
<td>-0.429</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Table 4.1: The correlation matrix.
CHAPTER 5 – THE “BUY AND HOLD” AND 1-YEAR TACTICAL STRATEGIES.

To understand concretely in which way the VIX instrument can improve the investor’s portfolio, that represent the essence of this work, I articulated 3 main strategies, with different length of time considered for the investment in VIX:

- Buy and hold strategy from January 2007 to June 2018;
- Tactical strategy annually from January 2007 to June 2018;
- Tactical strategy intra-monthly from January 2007 to June 2018.

For all the strategies, it is represented the comparison between the portfolio without the VIX instrument and the portfolio with the addition of the VIX instrument.

For all the strategies, I consider different measures of comparison:

- a graph of the efficient frontier with the portfolios that has to be compared;
- a graph of the total percentage return for each portfolio compared;
- a table of the most suitable indicators that allow me to compare the portfolios and to catch different information; the indicators listed in this table are the return, the standard deviation, the Coefficient of Variation, the Sharpe ratio, the Treynor ratio, the $M^2$ ratio, the Information ratio and the $\beta$ calculated with the CAPM model;
- a table with the most suitable downside measures of risk, that are the Value at Risk and Expected Shortfall, calculated with 95% level of confidence, and the Maximum Drawdown.

In this Chapter the first two strategies are developed.
5.1 “Buy and hold” strategy

Through this strategy the portfolio that started from January 2007 is maintained, without make any operations, lives until June 2018.

Supposing that the investment in the VIX instrument occurs in a static manner.

The static allocation means that the VIX instrument is put inside the portfolio that is held with those particular weights decided until the end of itself.

In practice, keeping the weights of mutual funds, composing the portfolio, equally proportional, I invest in the VIX instrument the fixed weight of 8%, percentage that derives from a hypothetical decrease of 1% from each mutual fund.

In Figure 5.1, the investor’s portfolio would be widely distant from the efficient frontier and it would give a very low return, 0.258% with a very high standard deviation, 8.033%. Although a slight improvement, in the standard deviation can be observed, since the initial portfolio shows a standard deviation of 10.506%, the return is worse including the VIX instrument, since the initial portfolio shows a return of 0.420%.

In Figure 5.2 the cumulative return of the initial portfolio and the portfolio with the introduction of the VIX instrument are represented; in few cases the portfolio with static percentage of the VIX instrument outperforms the initial portfolio.

In Table 5.1 the most suitable indicators for comparison the portfolio without and with the investment in the VIX instrument; except for the decrease in the standard deviation, with consequent reduction in the return, and the slightly better Information ratio highlighted with green indicator, the most efficient portfolio is the portfolio without the static adding of the VIX instrument.

![Efficient Frontier Jan 2007 – Jun 2018](image-url)
Figure 5.2: Comparison Portfolio Returns: static VIX.

![Comparison Portfolio Returns: static VIX](image)

<table>
<thead>
<tr>
<th>Indicators</th>
<th>PTF</th>
<th>PTF 8% STATIC VIX</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>μ</strong></td>
<td>0,420%</td>
<td>0,258%</td>
</tr>
<tr>
<td><strong>σ</strong></td>
<td>10,506%</td>
<td>8,033%</td>
</tr>
<tr>
<td>CV</td>
<td>24,987</td>
<td>31,173</td>
</tr>
<tr>
<td>Sharpe</td>
<td>0,033</td>
<td>0,023</td>
</tr>
<tr>
<td><strong>M^2</strong></td>
<td>0,233%</td>
<td>0,177%</td>
</tr>
<tr>
<td>Information</td>
<td>0,071</td>
<td>0,075</td>
</tr>
<tr>
<td>β</td>
<td>0,774</td>
<td>0,542</td>
</tr>
<tr>
<td>CAPM Portfolio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VaR</td>
<td>-5,653%</td>
<td>-6,766%</td>
</tr>
<tr>
<td>ES</td>
<td>-9,466%</td>
<td>-10,637%</td>
</tr>
<tr>
<td>Max DD</td>
<td>-39,480%</td>
<td>-44,233%</td>
</tr>
</tbody>
</table>

Table 5.1: Summary table of comparing indicators, from January 2007 to June 2018.

### 5.2 Tactical strategy: 1-year analysis.

In this Section, the analysis shifts the attention in a shorter length of time, 1 year. The reasons are first of all, the work of Dash and Moran in 2005 suggests that a tactical allocation of the VIX index is the best strategy comparing with no VIX or static VIX allocation; second, from the previous analysis, holding the VIX instrument in the portfolio for a very long-term period, is not the most efficient way to manage with this instrument; the history explains that it should be included with respect the time, the events occurred and the correlation with respect the other components of the portfolio.
The Section is divided in two parts that differ in the ex-ante and ex-post decision of how much VIX instrument allocate in the portfolio:

- ex-ante static allocation set at 8%;
- ex-post tactical allocation.

5.2.1 The static allocation of the VIX instrument.
The strategy functions in the following way: each year, at the beginning of the year, I rebalance the portfolio reporting the weights deriving from the past analysis January 2003 – December 2006 and from the characteristics of the investor. Therefore, at the beginning of each year the portfolio starts with the initial weights of the mutual funds of the investor’s portfolio.
I invest in the VIX instrument the fixed weight of 8%, percentage that derives from a hypothetical decrease of 1% from each mutual fund, each year.

Comparison measures
The Figure 5.4 presents the efficient frontier of the portfolio including the VIX instrument and the two portfolios: the portfolio without the VIX instrument and the portfolio with the static allocation at 8% of the VIX instrument, for each year.
The legend for reading the graphs of the comparison of the two portfolios is showed in Figure 5.3.
In Table 5.2 the most suitable indicators for comparison the portfolio without and with the investment in the VIX instrument, each year, are represented; only in 2 years the portfolio with static allocation of the VIX instrument at 8% results in better indicators with respect to the portfolio without the VIX instrument.
In Table 5.3 the indicators regarding the downside measures of risk, even for both portfolios each year, are represented; only in 2008, the portfolio with static allocation of the VIX instrument at 8% results in better VaR and Maximum Drawdown with respect to the portfolio without the VIX instrument.

Figure 5.3: Legend for the comparison of portfolios.
Figure 5.4: Comparison, based on the efficient frontier, among the portfolio without and with static allocation at 8% of the VIX instrument, each year, from 2007 to 2018.
<table>
<thead>
<tr>
<th>Period</th>
<th>Portfolio</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>CV</th>
<th>Sharpe</th>
<th>Treynor</th>
<th>$\mu^2$</th>
<th>Information</th>
<th>$\beta$</th>
<th>CAPM Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>Ptf</td>
<td>0.418%</td>
<td>7.991%</td>
<td>19.131</td>
<td>0.044</td>
<td>0.004</td>
<td>0.419%</td>
<td>-0.020</td>
<td>0.774</td>
<td></td>
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<tr>
<td></td>
<td>Ptf static VIX</td>
<td>0.807%</td>
<td>6.585%</td>
<td>8.164</td>
<td>0.112</td>
<td>0.014</td>
<td>0.611%</td>
<td>0.075</td>
<td>0.542</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf</td>
<td>-3.488%</td>
<td>17.479%</td>
<td>-5.012</td>
<td>-0.204</td>
<td>0.027</td>
<td>-1.160%</td>
<td>0.105</td>
<td>0.641</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-2.385%</td>
<td>12.230%</td>
<td>-5.127</td>
<td>-0.201</td>
<td>-0.045</td>
<td>-0.266%</td>
<td>-0.308</td>
<td>0.336</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>Ptf</td>
<td>2.712%</td>
<td>10.667%</td>
<td>3.934</td>
<td>0.248</td>
<td>0.085</td>
<td>1.746%</td>
<td>0.254</td>
<td>0.311</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>2.240%</td>
<td>8.871%</td>
<td>3.961</td>
<td>0.245</td>
<td>0.040</td>
<td>0.984%</td>
<td>0.283</td>
<td>0.227</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>Ptf</td>
<td>1.539%</td>
<td>7.133%</td>
<td>4.633</td>
<td>0.206</td>
<td>0.062</td>
<td>1.257%</td>
<td>0.649</td>
<td>0.238</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>1.296%</td>
<td>5.655%</td>
<td>4.364</td>
<td>0.217</td>
<td>0.023</td>
<td>0.905%</td>
<td>0.272</td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>Ptf</td>
<td>-0.453%</td>
<td>9.893%</td>
<td>-21.864</td>
<td>-0.053</td>
<td>-0.011</td>
<td>-0.191%</td>
<td>0.043</td>
<td>0.474</td>
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<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-0.409%</td>
<td>6.034%</td>
<td>-14.753</td>
<td>-0.079</td>
<td>-0.009</td>
<td>0.074%</td>
<td>-0.288</td>
<td>0.195</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>Ptf</td>
<td>0.827%</td>
<td>6.938%</td>
<td>8.389</td>
<td>0.109</td>
<td>0.023</td>
<td>0.454%</td>
<td>0.119</td>
<td>0.335</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>0.152%</td>
<td>6.091%</td>
<td>40.100</td>
<td>0.013</td>
<td>0.002</td>
<td>0.335%</td>
<td>-0.113</td>
<td>0.127</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>Ptf</td>
<td>0.644%</td>
<td>5.745%</td>
<td>8.927</td>
<td>0.100</td>
<td>0.014</td>
<td>0.283%</td>
<td>0.376</td>
<td>0.407</td>
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<tr>
<td></td>
<td>Ptf static VIX</td>
<td>0.086%</td>
<td>4.672%</td>
<td>54.202</td>
<td>0.003</td>
<td>0.000</td>
<td>0.307%</td>
<td>-0.233</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>Ptf</td>
<td>0.886%</td>
<td>4.434%</td>
<td>5.006</td>
<td>0.184</td>
<td>0.018</td>
<td>0.459%</td>
<td>0.319</td>
<td>0.462</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>0.631%</td>
<td>2.963%</td>
<td>4.693</td>
<td>0.189</td>
<td>0.010</td>
<td>0.829%</td>
<td>0.704</td>
<td>0.153</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>Ptf</td>
<td>0.963%</td>
<td>13.285%</td>
<td>81.490</td>
<td>0.007</td>
<td>0.001</td>
<td>0.288%</td>
<td>0.043</td>
<td>0.719</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>0.661%</td>
<td>11.417%</td>
<td>186.515</td>
<td>-0.001</td>
<td>0.000</td>
<td>0.295%</td>
<td>-0.053</td>
<td>0.455</td>
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</tr>
<tr>
<td>2016</td>
<td>Ptf</td>
<td>0.651%</td>
<td>5.688%</td>
<td>8.734</td>
<td>0.102</td>
<td>0.012</td>
<td>0.302%</td>
<td>0.086</td>
<td>0.484</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>0.318%</td>
<td>3.826%</td>
<td>12.029</td>
<td>0.065</td>
<td>0.005</td>
<td>0.479%</td>
<td>-0.199</td>
<td>0.254</td>
<td></td>
</tr>
<tr>
<td>2017</td>
<td>Ptf</td>
<td>0.716%</td>
<td>4.753%</td>
<td>6.634</td>
<td>0.136</td>
<td>0.006</td>
<td>0.081%</td>
<td>-0.207</td>
<td>1.055</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-0.017%</td>
<td>4.516%</td>
<td>-264.498</td>
<td>-0.019</td>
<td>-0.002</td>
<td>0.243%</td>
<td>-0.315</td>
<td>0.861</td>
<td></td>
</tr>
<tr>
<td>2018</td>
<td>Ptf</td>
<td>0.438%</td>
<td>4.819%</td>
<td>10.991</td>
<td>0.076</td>
<td>0.009</td>
<td>0.211%</td>
<td>0.347</td>
<td>0.388</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>0.459%</td>
<td>3.278%</td>
<td>7.134</td>
<td>0.119</td>
<td>0.007</td>
<td>0.631%</td>
<td>-0.132</td>
<td>0.217</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Summary table of comparing indicators, annually.

<table>
<thead>
<tr>
<th>Period</th>
<th>Portfolio</th>
<th>VaR</th>
<th>ES</th>
<th>Max DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>Ptf</td>
<td>-4.447%</td>
<td>-5.262%</td>
<td>-7.636%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-4.950%</td>
<td>-6.088%</td>
<td>-8.177%</td>
</tr>
<tr>
<td></td>
<td>Ptf</td>
<td>-12.617%</td>
<td>-13.752%</td>
<td>-37.832%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-12.614%</td>
<td>-13.380%</td>
<td>-35.991%</td>
</tr>
<tr>
<td>2009</td>
<td>Ptf</td>
<td>-3.033%</td>
<td>-4.483%</td>
<td>-37.590%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-4.049%</td>
<td>-5.637%</td>
<td>-39.070%</td>
</tr>
<tr>
<td>2010</td>
<td>Ptf</td>
<td>-3.445%</td>
<td>-4.630%</td>
<td>-17.765%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-4.747%</td>
<td>-6.022%</td>
<td>-20.574%</td>
</tr>
<tr>
<td>2011</td>
<td>Ptf</td>
<td>-6.222%</td>
<td>-8.411%</td>
<td>-16.323%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-7.308%</td>
<td>-9.755%</td>
<td>-20.571%</td>
</tr>
<tr>
<td>2012</td>
<td>Ptf</td>
<td>-3.271%</td>
<td>-4.502%</td>
<td>-9.543%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-4.956%</td>
<td>-6.452%</td>
<td>-15.254%</td>
</tr>
<tr>
<td>2013</td>
<td>Ptf</td>
<td>-2.780%</td>
<td>-3.876%</td>
<td>-9.993%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-4.432%</td>
<td>-5.488%</td>
<td>-16.485%</td>
</tr>
<tr>
<td>2014</td>
<td>Ptf</td>
<td>-2.216%</td>
<td>-2.663%</td>
<td>-8.789%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-3.267%</td>
<td>-3.837%</td>
<td>-15.574%</td>
</tr>
<tr>
<td>2015</td>
<td>Ptf</td>
<td>-5.857%</td>
<td>-7.206%</td>
<td>-15.818%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-6.583%</td>
<td>-8.072%</td>
<td>-22.107%</td>
</tr>
<tr>
<td>2016</td>
<td>Ptf</td>
<td>-3.055%</td>
<td>-4.245%</td>
<td>-16.764%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-4.078%</td>
<td>-5.843%</td>
<td>-23.121%</td>
</tr>
<tr>
<td>2017</td>
<td>Ptf</td>
<td>-1.991%</td>
<td>-2.509%</td>
<td>-10.138%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-3.171%</td>
<td>-3.847%</td>
<td>-17.218%</td>
</tr>
<tr>
<td>2018</td>
<td>Ptf</td>
<td>-3.076%</td>
<td>-3.685%</td>
<td>-9.597%</td>
</tr>
<tr>
<td></td>
<td>Ptf static VIX</td>
<td>-4.270%</td>
<td>-4.016%</td>
<td>-17.450%</td>
</tr>
</tbody>
</table>

Table 5.3: Summary table downside risk measure, annually.
5.2.2 The ex-post allocation of the VIX instrument.

Supposing to have a crystal ball, relying on the principles explained from the Markowitz theory and on the different events occurred in the financial market, I found, for each year, the best portfolio combination and the best amount of the VIX to be included. In this Section, I investigate how the portfolio would go if the VIX instrument had been added from the beginning, related to annual rebalances of the portfolio, from a “buy and hold” to a tactical portfolio management strategy.

Measures and observations.

Following, there are measures, with respect to each year, in terms of correlation of the VIX with respect all other mutual funds of the portfolio, risk/return of the portfolios, different risk indicators, portfolio weights, results of asset pricing models and downside measures of risk.

Correlations

The Table 5.4 shows the correlation between the VIX instrument and each mutual fund of the investor’s portfolio, listed for each year from January 2007 to June 2018. The theory stated that the VIX index is negatively correlated with the S&P 500 index; here, it can be observed that the VIX instrument is positively correlated only with the SISF – EURO Government Bond C Acc EUR mutual fund, unless in 2013 and 2015. It is, also, positively correlated with the BGF – Emerging Markets A2 EUR mutual fund in 2010, 2012, 2014 and 2017; then, with Janus Henderson – Balanced A EUR Acc (Hedged) and with Pictet-Water-R EUR in 2017. With all other mutual funds and for all other years, the VIX instrument is negatively correlated from a weak to a strong correlation. Therefore, the VIX instrument can be considered an appropriate element for diversification purposes, especially in turbulent periods, since it moves in the opposite direction compared to the other mutual funds of the portfolio.
Table 5.4: Correlations with the VIX instrument.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>-0.398</td>
<td>-0.134</td>
<td>-0.041</td>
<td>-0.100</td>
<td>-0.323</td>
<td>-0.653</td>
<td>-0.416</td>
<td>0.431</td>
</tr>
<tr>
<td>2008</td>
<td>-0.882</td>
<td>-0.357</td>
<td>-0.474</td>
<td>-0.516</td>
<td>-0.692</td>
<td>-0.765</td>
<td>-0.550</td>
<td>0.385</td>
</tr>
<tr>
<td>2009</td>
<td>-0.481</td>
<td>-0.123</td>
<td>-0.422</td>
<td>-0.101</td>
<td>-0.642</td>
<td>-0.407</td>
<td>-0.620</td>
<td>0.492</td>
</tr>
<tr>
<td>2010</td>
<td>-0.896</td>
<td>-0.206</td>
<td>-0.445</td>
<td>0.600</td>
<td>-0.492</td>
<td>-0.496</td>
<td>-0.643</td>
<td>0.525</td>
</tr>
<tr>
<td>2011</td>
<td>-0.817</td>
<td>-0.730</td>
<td>-0.450</td>
<td>-0.252</td>
<td>-0.704</td>
<td>-0.464</td>
<td>-0.846</td>
<td>0.173</td>
</tr>
<tr>
<td>2012</td>
<td>-0.618</td>
<td>-0.437</td>
<td>-0.083</td>
<td>0.564</td>
<td>-0.497</td>
<td>-0.353</td>
<td>-0.470</td>
<td>0.208</td>
</tr>
<tr>
<td>2013</td>
<td>-0.796</td>
<td>-0.112</td>
<td>-0.442</td>
<td>-0.064</td>
<td>-0.352</td>
<td>-0.292</td>
<td>-0.605</td>
<td>-0.432</td>
</tr>
<tr>
<td>2014</td>
<td>-0.847</td>
<td>-0.268</td>
<td>-0.694</td>
<td>0.466</td>
<td>-0.621</td>
<td>-0.408</td>
<td>-0.679</td>
<td>0.168</td>
</tr>
<tr>
<td>2015</td>
<td>-0.625</td>
<td>-0.533</td>
<td>-0.412</td>
<td>-0.031</td>
<td>-0.370</td>
<td>-0.350</td>
<td>-0.335</td>
<td>-0.130</td>
</tr>
<tr>
<td>2016</td>
<td>-0.832</td>
<td>-0.658</td>
<td>-0.646</td>
<td>-0.007</td>
<td>-0.751</td>
<td>-0.342</td>
<td>-0.637</td>
<td>0.009</td>
</tr>
<tr>
<td>2017</td>
<td>0.143</td>
<td>-0.329</td>
<td>0.182</td>
<td>0.267</td>
<td>-0.377</td>
<td>-0.277</td>
<td>-0.061</td>
<td>0.732</td>
</tr>
<tr>
<td>2018</td>
<td>-0.631</td>
<td>-0.558</td>
<td>-0.706</td>
<td>-0.225</td>
<td>-0.639</td>
<td>-0.913</td>
<td>-0.950</td>
<td>0.421</td>
</tr>
</tbody>
</table>

**Portfolio weights**

The graph, in Figure 5.5, shows the change in asset allocation through a compound ring that widens from the 2007 year in the center until the 2018 six months on the margins. With the red color is highlighted the change in the weights of the VIX instrument through years.

The Table 5.5 shows in detail the different weights given to different portfolio components for each year in the rebalancing activity. The weights are not given based on the Dash and Moran (2005) study but are set ex-post considering the efficiency of portfolios with respect to the related efficient frontiers and the constraints listed in the initial choice of the weights of the portfolio, excluding the VIX instrument:

- for the principle of the inclusion of all mutual funds in the portfolio, each mutual fund must be in the portfolio for a weight at least of 1%;
- for the principle of non-concentration, each mutual fund cannot exceed the weight of 25%.

The VIX index is weighted a lot in 2007 and 2008, years in which the financial crisis happened and the S&P 500 index expressed very negative results; then, it is weighted in a consistent way in 2011, due to the fall of stock markets in the United States, Middle East, Europe and Asia; finally, it is weighted also in a consistent way in the 6 months of 2018, since were months of diverging performance from the net positive performance in average to negative results for seven of eleven industry sectors.

In the remaining years, the VIX instrument is weighted in a range between 2% and 5%.
An exception takes place in 2017, in which the VIX instrument is weighted zero, primarily for very low levels of volatility during the year, considering also that the VIX index has reached its minimum 9.14 on 03 November. Then, for the synchronized world economic recovery bursts from a strong corporate earnings growth, stability given from elections in France, Germany and Japan and positive trend of technology, emerging markets and commodities sectors.

![Asset Allocation - Rebalance Jan 2007 - Jun 2018](image)

Figure 5.5: Asset Allocation – Rebalance Jan 2007 – Jun 2018.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Janus Henderson - Balanced A EUR Acc (Hedged)</td>
<td>9.00%</td>
<td>5.00%</td>
<td>5.00%</td>
<td>10.00%</td>
<td>5.00%</td>
<td>25.00%</td>
<td>20.00%</td>
<td>24.00%</td>
<td>10.00%</td>
<td>5.00%</td>
<td>10.00%</td>
<td>15.00%</td>
<td>5.00%</td>
</tr>
<tr>
<td>Pictet-Digital R in EUR</td>
<td>10.00%</td>
<td>5.00%</td>
<td>1.00%</td>
<td>8.00%</td>
<td>8.00%</td>
<td>1.00%</td>
<td>5.00%</td>
<td>24.00%</td>
<td>16.00%</td>
<td>30.00%</td>
<td>5.00%</td>
<td>10.00%</td>
<td>15.00%</td>
</tr>
<tr>
<td>Pictet-Water-R EUR</td>
<td>8.00%</td>
<td>5.00%</td>
<td>1.00%</td>
<td>5.00%</td>
<td>5.00%</td>
<td>1.00%</td>
<td>5.00%</td>
<td>10.00%</td>
<td>10.00%</td>
<td>7.00%</td>
<td>5.00%</td>
<td>10.00%</td>
<td>4.00%</td>
</tr>
<tr>
<td>BGF - Emerging Markets A2 EUR</td>
<td>15.00%</td>
<td>5.00%</td>
<td>15.00%</td>
<td>10.00%</td>
<td>15.00%</td>
<td>15.00%</td>
<td>15.00%</td>
<td>12.00%</td>
<td>20.00%</td>
<td>7.00%</td>
<td>15.00%</td>
<td>10.00%</td>
<td>5.00%</td>
</tr>
<tr>
<td>GS - Glo High Yield Pf E Cap EUR</td>
<td>9.00%</td>
<td>5.00%</td>
<td>1.00%</td>
<td>10.00%</td>
<td>5.00%</td>
<td>1.00%</td>
<td>5.00%</td>
<td>7.00%</td>
<td>5.00%</td>
<td>9.00%</td>
<td>5.00%</td>
<td>20.00%</td>
<td>3.00%</td>
</tr>
<tr>
<td>Carmignac Pfl Commodities A EUR Acc</td>
<td>14.00%</td>
<td>15.00%</td>
<td>1.00%</td>
<td>15.00%</td>
<td>20.00%</td>
<td>1.00%</td>
<td>3.00%</td>
<td>7.00%</td>
<td>5.00%</td>
<td>7.00%</td>
<td>25.00%</td>
<td>10.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>BL-Equities Europe B EUR Acc</td>
<td>22.00%</td>
<td>5.00%</td>
<td>1.00%</td>
<td>19.00%</td>
<td>15.00%</td>
<td>1.00%</td>
<td>25.00%</td>
<td>5.00%</td>
<td>12.00%</td>
<td>10.00%</td>
<td>5.00%</td>
<td>10.00%</td>
<td>10.00%</td>
</tr>
<tr>
<td>SISF - EURO Government Bond C Acc EUR</td>
<td>13.00%</td>
<td>10.00%</td>
<td>25.00%</td>
<td>20.00%</td>
<td>25.00%</td>
<td>25.00%</td>
<td>20.00%</td>
<td>9.00%</td>
<td>20.00%</td>
<td>20.00%</td>
<td>26.00%</td>
<td>15.00%</td>
<td>10.00%</td>
</tr>
<tr>
<td>VIX instrument</td>
<td>-</td>
<td>45.00%</td>
<td>50.00%</td>
<td>3.00%</td>
<td>2.00%</td>
<td>30.00%</td>
<td>2.00%</td>
<td>2.00%</td>
<td>2.00%</td>
<td>5.00%</td>
<td>4.00%</td>
<td>0.00%</td>
<td>28.00%</td>
</tr>
<tr>
<td>Total</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Table 5.5: Portfolio weights.
**Comparison measures**

The Figure 5.7 presents the comparison between the portfolio without the VIX instrument and the portfolio with the allocation deriving ex-post of the VIX instrument with respect to the efficient frontier, each year from 2007 to 2018; in each graph there is also the portfolio with static allocation at 8% of the VIX instrument.

The legend for reading the graphs of the comparison of the portfolios is showed in Figure 5.6. In Table 5.6 the most suitable indicators for comparison the portfolio without and with the investment in the VIX instrument, each year, are represented; for all years the portfolio with ex-post allocation of the VIX instrument results in better indicators with respect to the portfolio without the VIX instrument.

In Table 5.7 the indicators regarding the downside measures of risk, even for both portfolios each year, are represented; VaR and ES are better for the VIX included portfolio in 5 years on 11 with respect to the portfolio without the VIX instrument; the Maximum Drawdown is better for the VIX included portfolio in 7 years on 11. Also in Figure 5.8 is clear the better Maximum Drawdown of the VIX included portfolio.

---

**Figure 5.6: Legend for the comparison of portfolios.**
Figure 5.7: Comparison, based on the efficient frontier, among the portfolio without, the portfolio with ex-post allocation and the portfolio with static allocation at 8% of the VIX instrument, each year, from 2007 to 2018.
Table 5.6: Summary table of comparing indicators, annually.

<table>
<thead>
<tr>
<th>Period</th>
<th>Portfolio</th>
<th>VaR (%)</th>
<th>ES (%)</th>
<th>Max DD (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>Pf</td>
<td>-4,447%</td>
<td>-5,262%</td>
<td>-7,636%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-7,281%</td>
<td>-10,118%</td>
<td>-10,365%</td>
</tr>
<tr>
<td>2008</td>
<td>Pf</td>
<td>-12,617%</td>
<td>-13,752%</td>
<td>-37,832%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-9,342%</td>
<td>-10,702%</td>
<td>-17,751%</td>
</tr>
<tr>
<td>2009</td>
<td>Pf</td>
<td>-3,033%</td>
<td>-4,483%</td>
<td>-37,509%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-3,225%</td>
<td>-4,670%</td>
<td>-35,402%</td>
</tr>
<tr>
<td>2010</td>
<td>Pf</td>
<td>-3,445%</td>
<td>-4,630%</td>
<td>-17,765%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-3,689%</td>
<td>-4,753%</td>
<td>-16,989%</td>
</tr>
<tr>
<td>2011</td>
<td>Pf</td>
<td>-6,222%</td>
<td>-8,411%</td>
<td>-16,323%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-8,899%</td>
<td>-11,118%</td>
<td>-27,720%</td>
</tr>
<tr>
<td>2012</td>
<td>Pf</td>
<td>-3,271%</td>
<td>-4,502%</td>
<td>-9,543%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-2,741%</td>
<td>-4,148%</td>
<td>-7,256%</td>
</tr>
<tr>
<td>2013</td>
<td>Pf</td>
<td>-2,780%</td>
<td>-3,876%</td>
<td>-9,993%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-2,600%</td>
<td>-3,448%</td>
<td>-7,159%</td>
</tr>
<tr>
<td>2014</td>
<td>Pf</td>
<td>-2,216%</td>
<td>-2,663%</td>
<td>-8,789%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-2,193%</td>
<td>-2,627%</td>
<td>-7,037%</td>
</tr>
<tr>
<td>2015</td>
<td>Pf</td>
<td>-5,857%</td>
<td>-7,206%</td>
<td>-15,818%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-6,352%</td>
<td>-7,915%</td>
<td>-17,707%</td>
</tr>
<tr>
<td>2016</td>
<td>Pf</td>
<td>-3,055%</td>
<td>-4,245%</td>
<td>-16,764%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-3,115%</td>
<td>-4,550%</td>
<td>-21,776%</td>
</tr>
<tr>
<td>2017</td>
<td>Pf</td>
<td>-1,991%</td>
<td>-2,509%</td>
<td>-10,138%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-1,679%</td>
<td>-2,131%</td>
<td>-8,346%</td>
</tr>
<tr>
<td>2018</td>
<td>Pf</td>
<td>-3,076%</td>
<td>-3,685%</td>
<td>-9,597%</td>
</tr>
<tr>
<td></td>
<td>Pf ex-post VIX</td>
<td>-7,492%</td>
<td>-4,951%</td>
<td>-36,286%</td>
</tr>
</tbody>
</table>

Table 5.7: Summary table downside risk measure, annually.
Figure 5.8: Comparison Drawdown.

**Portfolio cumulative returns**

In Figure 5.9 the total return of the portfolio without and with ex-post allocation of the VIX instrument are compared. The major gain in terms of return is reached in the second half of the 2008; in general, the weights chosen on the basis of the Markowitz’ theory and on the macroeconomic events make the portfolio readier to face different market fluctuations.

Figure 5.9: Comparison Portfolio Returns: rebalances and VIX
Asset pricing models

The last more detailed analysis derives from asset pricing models’ variables. In the Table 5.8, in the period January 2007 – June 2018, show the CAPM results for both portfolios, in the Table 5.9, the F&F results for both portfolios.

The results for α are similar for both asset pricing models, that is are better for the initial portfolio; only in 3 years the ex-post portfolio performs better than the market, with respect to the portfolio without the VIX instrument. For β, the rebalanced and VIX including portfolio is less volatile than the reference market with respect β of the initial portfolio.

Table 5.8: CAPM results.

<table>
<thead>
<tr>
<th>Period</th>
<th>Ptf α</th>
<th>Ptf ex-post VIX β</th>
<th>Ptf</th>
<th>Ptf ex-post VIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>0.00229</td>
<td>0.02669</td>
<td>0.49346</td>
<td>-0.50009</td>
</tr>
<tr>
<td>2008</td>
<td>-0.00253</td>
<td>-0.03051</td>
<td>0.11112</td>
<td>-1.50583</td>
</tr>
<tr>
<td>2009</td>
<td>0.01832</td>
<td>0.01688</td>
<td>0.30638</td>
<td>0.24172</td>
</tr>
<tr>
<td>2010</td>
<td>0.01286</td>
<td>0.01289</td>
<td>0.18111</td>
<td>0.10528</td>
</tr>
<tr>
<td>2011</td>
<td>0.00142</td>
<td>-0.00605</td>
<td>0.16917</td>
<td>-0.91117</td>
</tr>
<tr>
<td>2012</td>
<td>0.00697</td>
<td>0.00535</td>
<td>0.27290</td>
<td>0.11833</td>
</tr>
<tr>
<td>2013</td>
<td>0.00326</td>
<td>0.00264</td>
<td>0.37320</td>
<td>0.17417</td>
</tr>
<tr>
<td>2014</td>
<td>0.00863</td>
<td>0.00811</td>
<td>0.39307</td>
<td>0.21507</td>
</tr>
<tr>
<td>2015</td>
<td>0.00585</td>
<td>0.00499</td>
<td>0.73199</td>
<td>0.57924</td>
</tr>
<tr>
<td>2016</td>
<td>0.00659</td>
<td>0.00576</td>
<td>0.33449</td>
<td>0.17826</td>
</tr>
<tr>
<td>2017</td>
<td>-0.00645</td>
<td>-0.00645</td>
<td>0.98375</td>
<td>0.98375</td>
</tr>
<tr>
<td>2018</td>
<td>0.00582</td>
<td>0.00720</td>
<td>0.32182</td>
<td>-0.20102</td>
</tr>
</tbody>
</table>

Table 5.9: F&F results.

<table>
<thead>
<tr>
<th>Period</th>
<th>Ptf α</th>
<th>Ptf ex-post VIX β</th>
<th>SMB</th>
<th>HML</th>
<th>RMW</th>
<th>CMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>-0.00959</td>
<td>-0.00435</td>
<td>0.54498</td>
<td>-0.39111</td>
<td>0.62217</td>
<td>1.76330</td>
</tr>
<tr>
<td>2008</td>
<td>-0.00252</td>
<td>-0.00737</td>
<td>0.11504</td>
<td>-1.07585</td>
<td>0.02273</td>
<td>-0.03264</td>
</tr>
<tr>
<td>2009</td>
<td>0.00854</td>
<td>0.00850</td>
<td>0.66739</td>
<td>0.63979</td>
<td>0.35122</td>
<td>0.38125</td>
</tr>
<tr>
<td>2010</td>
<td>0.01590</td>
<td>0.01510</td>
<td>0.16969</td>
<td>0.11222</td>
<td>0.12411</td>
<td>0.12580</td>
</tr>
<tr>
<td>2011</td>
<td>0.00042</td>
<td>-0.00042</td>
<td>0.21259</td>
<td>-0.88623</td>
<td>0.03842</td>
<td>-0.01528</td>
</tr>
<tr>
<td>2012</td>
<td>0.01252</td>
<td>0.01224</td>
<td>0.20695</td>
<td>0.15369</td>
<td>-0.42406</td>
<td>-0.39236</td>
</tr>
<tr>
<td>2013</td>
<td>0.00275</td>
<td>0.00282</td>
<td>0.37411</td>
<td>0.30662</td>
<td>0.23400</td>
<td>0.23630</td>
</tr>
<tr>
<td>2014</td>
<td>0.00738</td>
<td>0.00693</td>
<td>0.36446</td>
<td>0.27420</td>
<td>-0.00009</td>
<td>-0.02280</td>
</tr>
<tr>
<td>2015</td>
<td>0.00477</td>
<td>0.00476</td>
<td>0.85057</td>
<td>0.73551</td>
<td>0.75394</td>
<td>-0.00276</td>
</tr>
<tr>
<td>2016</td>
<td>0.00652</td>
<td>0.00539</td>
<td>0.23215</td>
<td>0.14583</td>
<td>0.20321</td>
<td>0.19273</td>
</tr>
<tr>
<td>2017</td>
<td>-0.00974</td>
<td>-0.00974</td>
<td>1.10172</td>
<td>1.10172</td>
<td>0.01037</td>
<td>0.01037</td>
</tr>
<tr>
<td>2018</td>
<td>-0.01147</td>
<td>0.00091</td>
<td>-0.24664</td>
<td>-0.30198</td>
<td>-0.40277</td>
<td>-0.16921</td>
</tr>
</tbody>
</table>
5.2.2.1 Tax implications: accrued and realized income.

As described above, the work does not consider cost implications since are different for each financial intermediary and during the years the indicator for expenses of mutual funds is changed and evolved.

Nevertheless, in this last Section, it is focused on another branch interesting for the investor: taxation.

Law references

In Italy, the taxation of a financial portfolio is complex and has changed over time. The tax rate has changed over time:

- 12.50% before 2011;
- 20.00% from the 1st January 2012;
- 26.00% from the 1st July 2014.

With Legislative Decree No. 461/1997 introduced three alternative regimes:

- in Article 5, the Tax Return Regime (TRR) or “regime della dichiarazione”;
- in Article 6, Non-Discretionary Mandate Regime (NDMR) or “regime del risparmio amministrato”;
- in Article 7, discretionary mandate or Portfolio Management Regime (PMR) or “regime del risparmio gestito”.

Under each article, the tax regime combined is the tax regime by default and the other two are optional.

“Under TRR each capital gain must be reported and taxed by the investor, whereas under the alternative two regimes the relevant Italian financial intermediary acts as a withholding agent for capital gain purposes, which eliminates residual reporting and tax obligations for the investors.”

According to the saving scheme adopted, declarative, administered or managed, the taxable amount changes.

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Major tax results
To evaluate the total tax impact, I followed 2 different calculations for the period January 2007 to December 2017.
For the SISF – EURO Government Bond C Acc EUR, I applied the tax rate of 12,50%, since on the government bond asset class are taxed with this tax rate.
For the 2014 I applied, for the entire year, the tax rate of 20%.

The accrued income
In the PMR, simply, each year, even if there are no movements of buying or selling, it must be calculated the difference between the value of the managed holding at the end of the year, gross of taxation, and the value of managed holding at the begging of the same year. The tax is applied to the entire result, if it is positive. If the result is negative, this capital loss can be reported to the following year, for a maximum of 4 years.
For the case of the annual tactical strategy, the total taxable amount is € 57.392,11, the tax is € 10.680,65 and the net result is € 46.711,46; the average tax rate applied is 18,61%. Accrued capital losses are € 0,00.
For the ex-post VIX included portfolio, the total taxable amount is € 193.658,14, the tax is € 24.257,41 and the net result is € 169.400,72; the average tax rate applied is 14,13%. Accrued capital losses are € -2.693,07.
It can be observed that the net result is more widely positive, and the average tax rate applied in the second case is lower than in the first case.

The realized income
In the NDMR, it must be calculated annually movements of each fund of the investor’s portfolio. The tax is applied to each capital gain deriving from selling of shares; the negative or capital losses are held for the following 4 years.
For the case of the annual tactical strategy, the tax is € 18.752,21 and the net result is € 53.371,68; the average tax rate applied is 26,00%. Accrued capital losses are € -967,34.
For the ex-post VIX included portfolio, the tax is € 30.357,63 and the net result is € 207.295,03; the average tax rate applied is 12,77%. Accrued capital losses are € -4.764,63.
As well as the accrued income results, it can be observed that the net result is more widely positive, and the average tax rate applied in the second case is lower than in the first case.
Consideration.

Depending on which type of saving scheme the investor chooses, tax bases and taxes are calculated in a different way, considering different elements.

With both calculation methods, accrued income and realized income, the portfolio that results in a higher total taxable amount in the ex-post VIX included one. Furthermore, it is also the portfolio with a lower average tax rate applied. Therefore, using ex-post allocation through rebalances and the VIX instrument can improve and takes advantages for tax purposes.

Regarding losses, for both calculation methods, accrued ones are higher for the ex-post VIX included portfolio, since it is used the tactical portfolio management strategy through buying and selling operations; therefore, it is more probable having losses when in the calculation method it is not possible to compensate capital gain and capital losses.
5.3 Conclusion.

From the portfolio analysis, multiple conclusions can be drawn.
For comparison purposes, recovering the Modern Portfolio Theory, with efficient frontier graphs, and considering that the investor’s portfolio is composed by only funds and not by securities or other form of investment, a point of view for comparison purposes, is the set of risk indicator, computed with the $\beta$ of each portfolio under the CAPM asset pricing model. Reminding one of the typical features of the VIX index, spikes up very fast but coming back goes gradually, it has an impact on the volatility of the including the VIX instrument in the portfolio, also downside measures of risk are considered: VaR and ES and the Maximum Drawdown.

The final result of the analysis deriving from the “buy and hold” strategy is definite: for the indicators measured, except for the Information ratios that is slightly better, the portfolio without addition of the VIX instrument is the preferable choice.

The final result for the analysis deriving from a tactical strategy, rebalancing the portfolio without VIX each year reporting the weights to the weights deriving from the past analysis of the period 2003-2006 and adding a static allocation at 8% of the VIX instrument, as the previous analysis, results in better indicators for the majority of the time, for the portfolio without the investment in the VIX instrument. Specifically, 7 years on 11 clearly makes to prefer the portfolio without any adding of the VIX index; for the rest of the year, some indicators are better for one portfolio or for the other one.

Remaining in the tactical strategy, the analysis continues in an opposite direction: from ex-ante decision to allocate static weight to the VIX instrument to the ex-post allocation of the VIX instrument in a portfolio totally revised. Percentage weights to the entire portfolio are set on the basis of Markowitz theory, considering the efficient frontier constructed each year with monthly values and maintaining the constraints set at the beginning for the mutual funds. The VIX instrument is weighted differently in correspondence of particular events happened, predominantly in downwards moves of the financial market. This means that the VIX instrument is used as hedging tool.

As the theoretical framework states, the VIX index is negatively correlated with the S&P 500 index, the most significative index, composed by a basket of United States stock; the VIX instrument in the real case study shows the only exception, that could be expected, in the predominant positive correlation with the European Government Bond mutual fund, since in general it has not linked to United States stock. But at the same time, it shows a negatively correlation with mutual funds belonging to asset classes that apparently are not linked to United States stock: for example, is predominantly negatively correlated with Emerging Markets mutual fund and totally with European Equities mutual fund. Therefore, under this point of
view, the VIX instrument could be a good diversifier tool in portfolio already constructed in a diversified way.

Since I’m ex-post, allocation of the weights is set to result in a better portfolio with respect to the portfolio without the VIX index, therefore the final result is definite: in the overall context, the portfolio with tactical annually rebalances and addition of the VIX instrument is better than the portfolio without it. For downside measures of risk, instead, it is more tolerable the portfolio without the VIX instrument; in the very difficult period of the financial crisis, the three measures of risk for the ex-post VIX included portfolio are much better. For this analysis, I include also a statistical point of view, considering results of CAPM and F&F 5-factor regressions, the portfolio without the VIX instrument has much higher ability to perform the market, but the ex-post VIX included portfolio has lower sensitivity to movements in the reference market. Including the VIX instrument in the portfolio, does not allow much to over-perform the reference market but to reduce its sensitivity towards market fluctuations. For a taxation point of view, the ex-post VIX included portfolio is the portfolio with more gains under accrued and realized methods for calculate taxation.

In a global perspective, considering measures explained above, the VIX element can allow to advantage in a portfolio constructed following the path of the case study of this work; an absolute principle to taking in mind is that whatever is the financial product, with the VIX index as underlying, it cannot be bought and left in a portfolio for the entire period of investment and it cannot allocate in a fixed static percentage. The investor has to invest in the VIX product in definite moment. It is difficult to understand when making the choice to add the VIX product in the portfolio, for this reason, I try to make a strategy that could allow to catch this moment in Part 3 of this work.
Part 3 – Ex-ante investing strategy in the VIX instrument.

In the Part 3, my analysis comes back, shifting the attention on the at the beginning of the investment, to understand when and how much invest, in percentage term, in the VIX instrument. Since a global and long-term strategy and a tactical one with static allocation of the VIX instrument, are proved to be not efficient ways to manage with this particular instrument, I enter in more details to discover a strategy that could take advantage of the improvement futures of this index.

For achieving this objective and for exploiting the point that the VIX index has to be included in the portfolio when is high, i.e. when volatility in the market is wide, for giving protection and diversification to the financial portfolio and therefore making the most of its characteristics and taking advantage by the peculiarity for which the VIX index goes up faster than when it goes down, I use moving averages to catch signals to buy or to sell. As a reference product, I consider the VIX instrument constructed in Chapter 4 of the Part 2 of this work.

The Part 3 is composed by 2 chapters:

- Chapter 1. The technical analysis, that explains the theoretical fundamentals on which is based and its relationship with moving averages.
- Chapter 2. The ex-ante tactical allocation strategy, that explains which aspects are considering for arriving to the implementation of it.
“The tactical allocation example suggested above is a starting point”\(^\text{19}\).

For taking advantages of the VIX index characteristics and properties, the best way to do it is to implement a tactical strategy, i.e. adjusting the portfolio after the analysis and observations of the market, changing the weights of asset classes.

Starting from the sentence above, reported from the Dash and Moran (2005) work, to improve and refine the strategy, I expanded and applied the technical analysis discipline linked to the concept of moving averages.

1.1 The technical analysis.

Technical analysis is a subject that needs to evaluate investments and try to catch trading opportunities by analyzing trends, mainly, from price movements, looking at graphics and implementing statistical-mathematical methods applied to historical series.

It was first introduced by Charles Henry Dow\(^\text{20}\), in the Dow theory in the late 1800s. With a series of observations on the behavior of financial markets through charts, technical analysis is born and is disseminated. Then, S. A. Nelson and William Hamilton improved the theory, presenting the modern technical analysis of today through books and newspaper articles.

The principles of the Dow theory

In summary, the Dow theory, that is a collection of assumptions and theorems can be explained in six points:

1. the market is composed by three trends: primary trend, that can be bullish or bearish, continues for a long-term period (more than a year); secondary trend, that correct the primary trend, for a medium-term period (1-3 months); minor trends for a short-period (from 1 day to 3 weeks);
2. the primary trend is divided in three phases: the first phase is the accumulation, in which only “big” and informed actors invest in aggressively way; the second is the public participation in which all come back to be confident; the third is the distribution in which also the world of retail investors starts to invest;
3. stock prices incorporate and reflect quickly all concerned information;


\(^{20}\) Charles Henry Dow (1851-1902), founder of the Wall Street Journal and inventor of the Dow Jones Industrial Average index. Information available on: [https://www.borsaitaliana.it/notizie/sotto-la-lente/analisi-tecnica-definizione.htm](https://www.borsaitaliana.it/notizie/sotto-la-lente/analisi-tecnica-definizione.htm) [Date of access: 28/11/2018].
4. it needs that the trend of one sector, i.e. the transportation one, is confirmed from the sectors linked to it, i.e. the industrial one; therefore, an increase in transportation securities symbolizes the increase in quotations of the industrial sector;

5. trends must be confirmed from traded volume: if the primary trend is bearish, volume should increase during market rises; if the primary trend is bullish, volume should increase during market declines;

6. trends exist until defined signals show that they are finished: an upward trend is defined by a series of increasing maximum and minimum points; if the trend records decreasing maximum and minimum points, the trend reverses in a downward trend.

1.2 The moving average: main characteristics.

Moving average or rolling average is a statistical indicator that allow to analyze values by creating a series of averages, taking subsets, of a prefixed size, from the entire dataset. It can be simple, cumulative or weighted.

**The Simple Moving Average (SMA)**

The Simple Moving Average (SMA) it is the arithmetic unweighted mean of a certain number of historical values. It is computed by taking values for a certain period of time and it is calculated the simple mean.

Despite some criticism to the application of this method, since it uses equal weights for past and recent data without distinction, I use this type of moving average because my example of technical analysis is focused on financial data and in particular on the VIX objective that does not represent a conventional instrument; the possible trend of the VIX index cannot be easily interpreting in advance.

Moreover, the variations in the mean resulting from this method are aligned with variations in the data, that is, they are not shifted in time. Graphically is really powerful to understand the trend of the element considered and capture possible trading signals.
Chapter 2. THE EX-ANTE TACTICAL ALLOCATION STRATEGY.

Based on the work of Dash and Moran (2005), I implement a tactical allocation strategy, in which given signals deriving from the technical analysis, the weight of the VIX instrument changes considering different aspects.

For understanding the potential of the VIX index inside a financial portfolio, differently from the aforementioned work, I do not consider the case in which the portfolio results with 0% of the VIX instrument; then, in the second scenarios, I introduce an ex-ante subjects for trying to anticipate the moment for investing in the VIX instrument: the technical analysis.

The methodology used is to construct efficient frontiers and the graph of the return of the portfolios, indicators based on risk/return relationship and downside measures of risk are considered.

For this tactical allocation strategy, I considered 2 different prefixed size for calculating the moving average: a long-term moving average of 6 months and a short-term moving average of 3 months, represented in Figure 2.1.

The background idea is the following: if the short-term moving average is higher than the long-term one, it means that the VIX instrument has to be included in the portfolio since it means that the volatility in the short is high. If the short-term moving average is lower than the long-term one, it means that the volatility in the short is low, so it is not appropriate to hold the VIX instrument in the portfolio.

![Moving averages: 3 and 6-month](image)

Figure 2.1: Moving averages: 3 and 6-month.
2.1 The definition of values and weights.

In the Figure 2.2, I consider the values of the VIX index and the values of the S&P 500 index, from January 2003 to December 2006; in the x axis there are the months, the y axis represents the values: in particular, the left y axis in the reference for the VIX index, the right side is the reference for the S&P 500 index.

When the S&P 500 index is not highly volatile and follows a positive trend, the VIX index stands in a range of values between 12 and 20; when the S&P 500 experienced large drops linked to high volatility, as in the case of the first months of 2003, the VIX index exceeds the value of 20, until arrive to a peak of about 31.

I observed, also, the same 48 monthly observations, about the prices of the VIX index, from January 2003 to December 2006, represented in Figure 2.3. The majority of the values, 81.25% that means 39 observations, stands between 12 and 20, so this can imply a “normal” path in which the VIX remains most of the time; 10.42% that means 5 observations are higher than 20, pointing out a wider distance of the single value from each other; 6.25% that means 3 observations are lower than 12, when there are no worries of downturns of the market.

Therefore, starting from this research and from the work of Pan and Poteshman (2006), who study, in particular, if the option trading contains information about future movements in underlying stock prices and arriving at a well-explained recent study conducted by authors belonging to the division of S&P Global in 2017, the VIX index can be enclosed in the price ranges, explained in Part 1 at the Chapter 3 of this work. In this last work, the authors refer to a study that compares the volatility changes in the S&P 500, when the VIX index level is high, S&P 500 variations are wide; if it is low, the contrary.

Figure 2.2: The VIX index and the S&P 500 index: January 2003 – December 2006.
Since the technical analysis starts from October 2006, for the dynamic allocation of the VIX instrument I recall the study conducted by Dash and Moran (2005) in which they set and decide the percentage weight to give to the VIX element in a range between 0% and 10%; the reason is that adding more weight, it would sacrifice the portfolio return, since the VIX index in the long-term reach an average around the zero. 

In my case, considering the logical of the VIX index level and the weight range set by Dash and Moran, I set the following weights:

- 2% when the VIX index, at the “BUY” signal, is “low”, i.e. lower than 12;
- 4% when the VIX index, at the “BUY” signal, is “normal”, i.e. higher than 12 and lower than 20;
- 10% when the VIX index, at the “BUY” signal, is “high”, i.e. higher than 20.

The reason of this division of weights is that when I receive the “BUY” signal, I have to invest at least a small amount in the VIX instrument; when the correspondence VIX index level is “high”, the weight has to be the maximum to really exploit the improvement to adding a VIX instrument in the portfolio.

2.2 The application: buying and selling signals.

The graph deriving by the execution of some aspects of the technical analysis highlights signals that allow me to understand when it is the moment to enter in the investment of the VIX instrument on the basis of VIX index prices.

As measurements about return and standard deviations are made ex-post, I consider only signals that allow me to hold the VIX instrument in the portfolio for at least 3 months.

In Figure 2.4, 8 periods are identified and for each of them I calculated the return and the risk of the portfolio, comparing them with the portfolio without the investment in the VIX instrument.
In the same figure, the indexes of the main asset classes are represented for understand the fluctuations with respect to the 8 periods, in the right side of the y axis. The indexes represented are the S&P 500 index for the U.S. stock asset class, the MSCI Emerging Markets index for the emerging market stock asset class and the EURO STOXX 50 index for the European stock asset class.

At the moment of a “BUY” signal, I observe the price of the VIX index and decide its weight regarding the rule set previously. The weights attributed to each period are listed in Table 2.1.

<table>
<thead>
<tr>
<th>Period</th>
<th>VIX index</th>
<th>VIX weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 2008 - Apr 2008</td>
<td>26,20</td>
<td>10%</td>
</tr>
<tr>
<td>Aug 2008 - Jan 2009</td>
<td>20,65</td>
<td>10%</td>
</tr>
<tr>
<td>May 2010 - Aug 2010</td>
<td>32,07</td>
<td>10%</td>
</tr>
<tr>
<td>Jul 2011 - Nov 2011</td>
<td>25,25</td>
<td>10%</td>
</tr>
<tr>
<td>Jun 2013 - Oct 2013</td>
<td>16,86</td>
<td>4%</td>
</tr>
<tr>
<td>Aug 2014 - Mar 2015</td>
<td>11,98</td>
<td>2%</td>
</tr>
<tr>
<td>Jul 2015 - Oct 2015</td>
<td>12,12</td>
<td>4%</td>
</tr>
<tr>
<td>Dec 2017 - May 2018</td>
<td>11,04</td>
<td>2%</td>
</tr>
</tbody>
</table>

Table 2.1: Weights allocation of the VIX instrument based on the VIX index price at the moment of the “BUY” signal.
Figure 2.4: Technical analysis: selected periods and comparison with the main indexes.
2.3 Measures and observations.

Following, there are measures, with respect to the 8 selected periods, in terms of efficient frontier, risk/return of the portfolios, different risk indicators, downside measures of risk.

Comparison measures

The Figure 2.6 presents the comparison between the portfolio without the VIX instrument and the portfolio with ex-ante tactical allocation of the VIX instrument with respect to the efficient frontier, each of the 8 periods selected.

The legend for reading the graphs of the comparison of the portfolios is showed in Figure 2.5.

In Table 2.2 the most suitable indicators for comparison the portfolio without and with the investment in the VIX instrument, for each of the 8 periods selected, are represented. With respect to the indicators considered in Chapter 5 of Part 2 of this work, at this point I considered only risk indicators that can have a certain degree of significance; regressions models could give distorted results since for each of the 8 periods there are few values to consider.

The result is that in 5 periods on 8 the portfolio including the VIX instrument is better than the portfolio without the VIX instrument.

In Table 2.3 the indicators regarding the downside measures of risk, even for both portfolios each of the 8 periods, are represented; VaR and ES are more tolerable for the portfolio without the VIX instrument, in all the 8 periods selected; the Maximum Drawdown is better for the VIX included portfolio in 2 periods. Also, in Figure 2.7 is clear the comparison of the Maximum Drawdown of the two portfolios.

Figure 2.5: Legend for comparison of efficient frontiers.
Figure 2.6: Comparison among efficient frontiers, portfolios, equally weighted portfolios and Sharpe portfolios before and after tactical VIX allocation in 8 periods selected.
Table 2.2: Summary table of comparing indicators, 8 periods selected.

<table>
<thead>
<tr>
<th>Period</th>
<th>Portfolio</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>CV</th>
<th>Sharpe</th>
<th>$M^2$</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 2008 - Apr 2008</td>
<td>Ptf</td>
<td>0.221%</td>
<td>13.429%</td>
<td>60.832</td>
<td>0.016</td>
<td>0.295%</td>
<td>-0.073</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-0.309%</td>
<td>10.669%</td>
<td>-34.478</td>
<td>-0.029</td>
<td>0.120%</td>
<td>-0.174</td>
</tr>
<tr>
<td>Aug 2008 - Jan 2009</td>
<td>Ptf</td>
<td>-5.167%</td>
<td>19.025%</td>
<td>-3.682</td>
<td>-0.272</td>
<td>-2.074%</td>
<td>0.417</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-2.089%</td>
<td>10.127%</td>
<td>-4.848</td>
<td>-0.206</td>
<td>-1.500%</td>
<td>5.469</td>
</tr>
<tr>
<td>May 2010 - Aug 2010</td>
<td>Ptf</td>
<td>0.311%</td>
<td>2.291%</td>
<td>7.358</td>
<td>0.136</td>
<td>0.934%</td>
<td>-0.146</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>0.337%</td>
<td>4.541%</td>
<td>13.480</td>
<td>0.074</td>
<td>0.523%</td>
<td>-0.065</td>
</tr>
<tr>
<td>Jul 2011 - Nov 2011</td>
<td>Ptf</td>
<td>-1.306%</td>
<td>14.793%</td>
<td>-11.237</td>
<td>-0.088</td>
<td>-0.666%</td>
<td>0.183</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>0.266%</td>
<td>6.009%</td>
<td>22.630</td>
<td>0.044</td>
<td>0.473%</td>
<td>-1.047</td>
</tr>
<tr>
<td>Jun 2013 - Oct 2013</td>
<td>Ptf</td>
<td>1.326%</td>
<td>4.074%</td>
<td>3.071</td>
<td>0.326</td>
<td>1.151%</td>
<td>0.421</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>0.945%</td>
<td>2.688%</td>
<td>2.843</td>
<td>0.352</td>
<td>1.241%</td>
<td>2.338</td>
</tr>
<tr>
<td>Aug 2014 - Mar 2015</td>
<td>Ptf</td>
<td>1.796%</td>
<td>6.386%</td>
<td>3.555</td>
<td>0.281</td>
<td>0.772%</td>
<td>0.495</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>1.793%</td>
<td>6.320%</td>
<td>3.252</td>
<td>0.284</td>
<td>0.779%</td>
<td>0.503</td>
</tr>
<tr>
<td>Jul 2015 - Oct 2015</td>
<td>Ptf</td>
<td>-0.966%</td>
<td>18.496%</td>
<td>-19.144</td>
<td>-0.052</td>
<td>-0.404%</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-0.887%</td>
<td>15.509%</td>
<td>-17.494</td>
<td>-0.057</td>
<td>-0.441%</td>
<td>0.033</td>
</tr>
<tr>
<td>Dec 2017 - May 2018</td>
<td>Ptf</td>
<td>0.601%</td>
<td>7.336%</td>
<td>12.209</td>
<td>0.082</td>
<td>0.258%</td>
<td>0.188</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>0.608%</td>
<td>6.715%</td>
<td>11.043</td>
<td>0.091</td>
<td>0.290%</td>
<td>0.228</td>
</tr>
</tbody>
</table>

Table 2.3: Summary table downside risk measure, 8 periods selected.

<table>
<thead>
<tr>
<th>Period</th>
<th>Portfolio</th>
<th>VaR</th>
<th>ES</th>
<th>Max DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 2008 - Apr 2008</td>
<td>Ptf</td>
<td>-5.024%</td>
<td>-5.728%</td>
<td>-16.643%</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-7.111%</td>
<td>-7.999%</td>
<td>-16.090%</td>
</tr>
<tr>
<td>Aug 2008 - Jan 2009</td>
<td>Ptf</td>
<td>-15.146%</td>
<td>-15.698%</td>
<td>-37.300%</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-15.161%</td>
<td>-16.334%</td>
<td>-32.666%</td>
</tr>
<tr>
<td>May 2010 - Aug 2010</td>
<td>Ptf</td>
<td>-2.133%</td>
<td>-2.436%</td>
<td>-12.777%</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-5.678%</td>
<td>-6.595%</td>
<td>-14.145%</td>
</tr>
<tr>
<td>Jul 2011 - Nov 2011</td>
<td>Ptf</td>
<td>-10.231%</td>
<td>-11.138%</td>
<td>-16.225%</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-12.943%</td>
<td>-15.061%</td>
<td>-19.243%</td>
</tr>
<tr>
<td>Jun 2013 - Oct 2013</td>
<td>Ptf</td>
<td>-4.033%</td>
<td>-4.759%</td>
<td>-9.713%</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-5.566%</td>
<td>-6.450%</td>
<td>-12.749%</td>
</tr>
<tr>
<td>Aug 2014 - Mar 2015</td>
<td>Ptf</td>
<td>-1.008%</td>
<td>-1.176%</td>
<td>-5.357%</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-1.442%</td>
<td>-1.645%</td>
<td>-6.969%</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-8.939%</td>
<td>-9.477%</td>
<td>-18.206%</td>
</tr>
<tr>
<td>Dec 2017 - May 2018</td>
<td>Ptf</td>
<td>4.061%</td>
<td>-4.682%</td>
<td>-9.554%</td>
</tr>
<tr>
<td></td>
<td>Ptf ex-ante VIX</td>
<td>-4.262%</td>
<td>-4.902%</td>
<td>-10.957%</td>
</tr>
</tbody>
</table>
2.4 Cumulative return of the portfolio

In Figure 2.8 the total return of the portfolio without and with ex-ante tactical allocation of the VIX instrument are compared. The cumulative return of the initial portfolio and the portfolio with a tactical introduction of the VIX instrument are represented; in cases of portfolio downturns, the investment in the VIX instrument, at the moment of the “BUY” signal, enhances always the performance with respect to the portfolio without the VIX instrument; in some cases of portfolio upwards, instead, return is primarily lower but this is a consequence of lower standard deviations.

Figure 2.8: Comparison Portfolio Returns: tactical VIX.
2.5 Comparison of results of the three strategies.

In Table 2.4 the results of each year are represented: for the portfolio annual rebalances reporting the weights attributed after the past analysis 2003-2006; for the portfolio with the 8% static VIX instrument annual rebalances reporting to the weights deriving from the static allocation of the VIX instrument; for the portfolio with ex-post weights annual rebalancing to the most efficient weights able to exploit the VIX properties.

The highest total result would have been of the portfolio with ex-post allocation of the VIX instrument; comparing only the two ex-ante portfolios, the static one is the best.

In Figure 2.9 the same annual gains are showed in a line graph.

<table>
<thead>
<tr>
<th>Period</th>
<th>Ptf</th>
<th>Ptf static VIX</th>
<th>Ptf ex-post VIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>4.594,21</td>
<td>9.257,80</td>
<td>33.453,88</td>
</tr>
<tr>
<td>2008</td>
<td>-23.119,87</td>
<td>-5.904,08</td>
<td>114.487,70</td>
</tr>
<tr>
<td>2009</td>
<td>22.042,89</td>
<td>18.593,55</td>
<td>17.524,43</td>
</tr>
<tr>
<td>2010</td>
<td>19.142,36</td>
<td>12.483,57</td>
<td>12.374,13</td>
</tr>
<tr>
<td>2011</td>
<td>-21.896,99</td>
<td>-2.005,83</td>
<td>524,83</td>
</tr>
<tr>
<td>2012</td>
<td>8.176,11</td>
<td>3.022,11</td>
<td>3.109,46</td>
</tr>
<tr>
<td>2013</td>
<td>1.409,18</td>
<td>2.168,36</td>
<td>2.870,10</td>
</tr>
<tr>
<td>2014</td>
<td>3.375,74</td>
<td>5.564,17</td>
<td>2.551,80</td>
</tr>
<tr>
<td>2015</td>
<td>-12.332,32</td>
<td>909,90</td>
<td>890,20</td>
</tr>
<tr>
<td>2016</td>
<td>12.631,94</td>
<td>5.201,72</td>
<td>3.050,99</td>
</tr>
<tr>
<td>2017</td>
<td>-3.857,50</td>
<td>5.945,99</td>
<td>6.323,09</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10.165,75</strong></td>
<td><strong>55.237,26</strong></td>
<td><strong>197.160,60</strong></td>
</tr>
</tbody>
</table>

Table 2.4: Annual cumulative and total gains.

Figure 2.9: Annual cumulative gains.
In the Figure 2.10 cumulative returns in monetary term, €, are showed for the entire period, from January 2007 to June 2018.

The portfolio cumulative return is represented by the blue line, the portfolio with static 8% allocation of the VIX instrument is the green line, the tactical allocation made annually ex-post is the yellow line and the tactical allocation deriving from technical analysis is the red line.

The ex-post allocation is clearly the best, with a final gain of 366,225,32 €, but it is only indicative of how much the investor could gain if he knew what would happen before the time. Comparing the two ex-ante strategies, I can state that the intra-month tactical allocation is better than the portfolio itself without the VIX instrument and also than the static 8% allocation of the VIX instrument.

The amount reached, and the relative percentages achieved at the end of the analysis period, related to the initial capital invested, are showed in Table 2.4.

<table>
<thead>
<tr>
<th>Value</th>
<th>Date</th>
<th>Initial value (€)</th>
<th>Final value (€)</th>
<th>Final gain (€)</th>
<th>Final gain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ptf</td>
<td>31/01/07</td>
<td>80.000,00</td>
<td>133.353,06</td>
<td>53.353,06</td>
<td>66,69%</td>
</tr>
<tr>
<td></td>
<td>29/06/18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Static 8% VIX</td>
<td>29/06/18</td>
<td>80.000,00</td>
<td>109.700,21</td>
<td>29.700,21</td>
<td>37,13%</td>
</tr>
<tr>
<td>Tactical VIX</td>
<td>29/06/18</td>
<td>80.000,00</td>
<td>176.207,22</td>
<td>96.207,22</td>
<td>120,26%</td>
</tr>
</tbody>
</table>

Table 2.5: Initial and final value of the portfolio regarding static and tactical allocation of the VIX instrument.
Figure 2.10: Cumulative returns of the portfolio.
2.6 Considerations.
This Part of the work has the objective to show one of the possible strategies to try to anticipate fluctuations of the VIX instrument and exploit it to enhance return of the investment.
In the tactical strategy, thanks to the support of statistical and mathematical concepts, properties and characteristics of the VIX index have been exploited; it is a strategy that can be used for any other VIX index financial product.
The final result with an allocation founded on past studies and on “BUY” signals deriving from the technical analysis, for the majority of the periods selected, makes that is better this portfolio with respect to the portfolio without the VIX instrument. Therefore, the way in which I follow the strategy, that is when I receive “BUY” signals the first thing that I see is the level of the VIX index, can be considered a good way to work for trying to anticipate the moment and the amount of investing in the VIX instrument. More specifically, the decision to allocate more or less percentage weight in the VIX, follows the logic of the predictive property of the VIX index; when the VIX level is high this means that market operators attend high volatility of the S&P 500 index in the short term, therefore the investor needed to have an higher amount of the VIX in the portfolio. The contrary happens if the VIX level is low. Making calculations with the three percentage weights set, the allocation made by looking at the VIX index level is proved the best in 5 period on the 8 selected.
The strategy exposed can be applied in every period of time and for every temporal length.
Nowadays, the last signal of October 2018 is the “BUY” one; in fact, the last part of 2018 year it has been characterized by high volatility that can be conducted to the important rise of Treasury returns, put in place by the Federal Reserve in USA, to stop the monetary stimulus plan. But this has penalized the stock market because the market thinks that investors could prefer government bonds with respect to stock asset class. The volatility of the financial market increased and, therefore, it is correct having the VIX index, in any forms, in the investment portfolio.
Conclusion

The work, through the theory regarding the financial portfolio construction and the VIX index, tries to achieve the objective to analyze and represent the case of an investment done by a real investor and to extend results given by past studies.

To construct the financial portfolio, I recalled the Markowitz’ contribution to the Modern Portfolio Theory; in the financial world, he has introduced milestones regarding the following concepts: maximize return, minimize risk, correlations, diversification, efficient frontier. Today, with the developing of the world of saving, limits have been discovered: one for all, the limit represented from the generation of extreme solution when it is implemented the Markowitz’ optimization, that allow to maximize expected returns and minimize risk. In fact, in the work, the most efficient portfolio, with its constraints, does not lie on the efficient frontier. Nevertheless, the entire Markowitz’ theory has been a revolution in the investment process and in the investors’ mind; thanks to the feasible comprehension and logical reasoning, it has been created a new way to approach to the portfolio management. The investor’s way of thinking has changed, and, in my opinion, it is yet an efficient help and support for the meeting between the financial world and the single subject that wants to invest in this world.

The financial market is full of financial instruments and financial products; the investor must be protected and informed about risks and possible gains. Therefore, based on the place at the center the investor, I go through the volatility argument, element of not immediate comprehension, using a financial instrument: the VIX index.

The work has been articulated in three strategy that differs in terms of technique and length of time considered, from a global and a long-term perspective, buy and hold strategy, to more detailed and short-term analysis, intra-monthly strategy, passing through the annual analysis. The VIX is included in different way following the logic through which the decision is made ex-ante the life of the portfolio starts and in one case ex-post knowing all events occurred.

The choice of the weight of the VIX instrument also passes from a static vision to a dynamic one: in the static vision, the VIX instrument is simply something that derives from the mutual funds already present in the portfolio; I suppose to decrease each fund of 1% and adding 8% to the VIX instrument. After that, ex post, the weight of the VIX instrument derives from the best combination of asset inside the portfolio that makes that the particular weight of the VIX, accompanied by rebalances in the rest of the mutual funds, is the most efficient one to enhance the portfolio return, standard deviation and the series of indicators needed to make a comparison analysis with the portfolio without the VIX adding.

As past studies demonstrate, it was very probable that the VIX index could enhance a financial portfolio; what is not yet so much explored is concerning about the decision on when and for which amount of investment to make on a VIX product. Therefore, recalling the past studies
and the history of the VIX index, adding some new elements, the weight of the VIX instrument is the result of a reasoning. In the more detailed view of when and how invest in the VIX instrument are included: the range of allocation weight set by the work of Dash and Moran (2005), a strategy of technical analysis that consider short and long-term moving averages, through which their comparison produces signals that indicate the moment in which invest in the VIX instrument, the value of the VIX index to understand when the signals arrived, which amount in percentage term is the more appropriate amount investable in the VIX instrument.

The result, deriving from multiple comparison indicators, from ratios calculated only with mean and standard deviation to ratios that consider mean and standard deviation of the benchmark and downside measures of risk, starting from the most global perspective, is that the portfolio without the investment of the VIX instrument is better than a portfolio with a static investment held for 11 years and 6 months.

In the intermediate analysis that consider indicators calculated each year, the same result of the previous analysis is reached, with only some ratios exceptionally better for the portfolio with the static VIX instrument. The ex-post allocation instead results better than the portfolio without the VIX instrument and for the majority of the years also better of the portfolio with static investment in the VIX instrument.

In the last and the more detailed analysis, for the majority of the cases, the predictive property of the VIX index is demonstrated. It can be stated, also, that adding the VIX instrument with a tactical strategy, following signals that highlight the moment of buying or selling, can make a portfolio return more stable and decreases its standard deviation.

Finally, I can state that the result of the work made by Dash and Moran is compliant also for my case study because, comparing the cumulative return of the portfolio in €, the short-term tactical strategy combined with dynamic allocation of the VIX, is the best method to try to anticipate ex-ante when and how much VIX adding to a financial portfolio to reach enhanced performances.

Not only, the theory is confirmed, but in in some cases, the mathematical, statistical and econometric results of my work have been richer with respect many studies conducted on the VIX index and on the investment on it, selected and listed in the Chapter 3 of the Part 1 of this work. In particular, it is verified that adding a portion of the VIX index between 0% and 10% is a benefit tool (Dash, Moran – 2005) also for the portfolio explained in the case study and this small percentages can reduce the volatility of portfolio itself (Black – 2006). In the ex-post strategy of the work this is true but only in normal conditions of the market; in crisis or downturn moves, the VIX instrument should be present in the portfolio in a higher percentage for being a benefit tool.
It is also stated that a VIX index long position, for example, represented by buying an ETF based on the VIX index, in a stock portfolio based on S&P 500 index can allow to diversification, reducing the risk (Daigler, Rossi – 2006). In the work, this is true also for other asset classes different from the stock, based on distinct sectors and geographical areas; this is the case of the negative correlation and the effective diversification of the VIX instrument with the Emerging Markets and European Equity mutual funds.

The instrument used in the work, that is referred to the VIX index, is not a conventional financial product. First, because it is not known from the majority of the public of the investors; second, because it’s the own VIX index to not be of simple intuition.

This is the reason of the combination of the two parts: one part that aims to be close to the investor’s world, the Markowitz’ theory and the developed concepts, and the other part near only to the financial world, the meaning and the use of the VIX index following a hierarchical scheme from a passive long-term analysis to a tactical short-term analysis.

I can complete the work with two considerations: one is technical, since I can conclude the tactical allocation of the VIX instrument through the use of technical analysis can enhance performance of the portfolio, primarily, in downturns of the elements of the portfolio itself and, in general, of the market; the other one is human, since explaining the complex VIX index through feasible comprehensive concepts, in my opinion, it is a good-natured way to expand the knowledge of financial tools that can generate improvements to investors’ portfolios.


Websites


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